EE140 HW3 Solutions

In general, your answers don't need more than 1 sigfig

1)

\[
\begin{align*}
V_i & = 10 \text{V} \\
C & = 10 \text{pF} \\
V_o & = \frac{V_i}{100 \text{pF}} \\
\mu & = \frac{20 \mu A}{V^2} \\
W & = 10^4 \\
\lambda & = 0.01 \text{V} \\
V_{th} & = 1 \text{V}
\end{align*}
\]

a) \( I_D = \frac{V_{DD} - V_o}{R} \)

\[
I_D = \frac{10 - 9}{10 \text{k}} = 0.1 \text{mA}
\]

\( I_D \) range: 0.1 mA to 0.9 mA

+1 for \( I_D=f(V_o) \) equation

b) \( I_O = \frac{\mu C}{L} \frac{W}{2} (V_{os} - V_t)^2 \)

\( I_O \) at 0.1 mA:

\[
0.1 \text{mA} = \frac{20 \mu A}{2V^2} 10^4 (V_{os} - 1)^2 \\
V_{os} = 1.032 \text{mV} \\
V_{ov} = 32 \text{mV}
\]

\( I_O \) at 0.9 mA:

\[
0.9 \text{mA} = \frac{20 \mu A}{2V^2} 10^4 (V_{os} - 1)^2 \\
V_{os} = 1.095 \text{V} \\
V_{ov} = 95 \text{mV}
\]

\[ \Delta V_{ov} = 63 \text{mV} \]

+1 each for high and low \( V_{ov} \) and \( V_{os} \)

+1 for \( gm=f(V_o) \)

+1 for \( ro=f(V_o) \)

c) \( gm = \sqrt{2 \mu C} \frac{W}{L} \left( \frac{V_{DD} - V_o}{R} \right) \)

\( r_o = \frac{1}{\lambda I_D} = \frac{R}{\lambda (V_{DD} - V_o)} \)

+1 for \( gm=f(V_o) \)

+1 for \( ro=f(V_o) \)
(d) \[ A_v = -g_m r_o = \frac{2}{\lambda V_{ov}} \quad V_{ov} = \frac{2I_D}{\sqrt{\frac{2I_D}{V_{DD} - V_o}} \frac{1}{V R}} \]

\[ A_v = \frac{2}{\sqrt{\frac{2I_D}{V_{DD} - V_o}} \frac{1}{V R}} \]

+1 for an Av equation and +1 if it is a function of Vo

(e) \[
\begin{array}{cccccccc}
V_o & I_D & g_m & r_o & A_v & W_p & W_a \\
2V & 100\mu A & 6.3mS & 1M\Omega & 63 \% & 1Mrad/s & 63 Mrad/s \\
6V & 400\mu A & 12.6mS & 250k\Omega & 126 \% & 1Mrad/s & 126 Mrad/s \\
1V & 900\mu A & 18.9mS & 111k\Omega & 189 \% & 1Mrad/s & 189 Mrad/s \\
\end{array}
\]

+0.5 per entry, within ~10-20% is fine, 1 sig fig is sufficient

(f) \[ Z = 1\mu s \]

+1 per plot, ns scale should be ~flat, us scale should look exponential and end at 0.63 of the final value, ms scale should look like a step
\[ V_{dd} \downarrow \]
\[ R_L \downarrow \]
\[ -1 \downarrow \]

a) \[ R_L = R_o = \frac{1}{\lambda I_D} = \frac{1}{\frac{1}{V_{dd}} (V_{dd} - V_c)} \]

\[ \frac{10}{V_{dd} - 1} = 1 \quad \text{so} \quad V_{dd} = 11 \text{V} \]

b) If \( V_{dd} = 2 \text{V} \) and \( V_c = 1 \text{V} \)

\[ R_o = \frac{1}{\lambda I_D} = \frac{1}{0.1 \left( \frac{2-1}{R_L} \right)} = 10R_L \]

So we should approximate that \( R_o = R_L \).

\[ 10R_L \parallel R_L = \frac{10R_L}{11} \approx 0.9R_L \quad \text{Approximation is off } \approx 10\% \]

c) \[ A_V = -g m R_L \]

\[ g m = \frac{2I_D}{V_{dd}} \quad I_D = \frac{2-1}{R_L} = \frac{1}{R_L} \]

\[ A_V = -\frac{2}{V_{dd}} \cdot R_L \]

\[ \boxed{A_V = -\frac{2}{V_{dd}}} \]
(3) $A_v = 100$

$GBW = 10 MHz \cdot 20 = 200 MHz$  

$f_p$ has gain of 100: $\frac{200 MHz}{100} = 2 MHz$  

$f_u$ has gain of 1: $\frac{200 MHz}{1} = 200 MHz$

(4)

<table>
<thead>
<tr>
<th>$A_v$</th>
<th>$w_p$</th>
<th>$w_n$</th>
<th>$g_m$</th>
<th>$r_o$</th>
<th>$C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1M</td>
<td>100M</td>
<td>$10^{-4}$</td>
<td>$10^6$</td>
<td>1p</td>
</tr>
<tr>
<td>200</td>
<td>10M</td>
<td>2G</td>
<td>$2 \cdot 10^{-3}$</td>
<td>100k</td>
<td>1p</td>
</tr>
<tr>
<td>100</td>
<td>10M</td>
<td>1G</td>
<td>$10^{-4}$</td>
<td>1M</td>
<td>100f</td>
</tr>
<tr>
<td>$10^6$</td>
<td>10</td>
<td>10M</td>
<td>$10^{-4}$</td>
<td>$10^{10}$</td>
<td>10p</td>
</tr>
</tbody>
</table>

+2 for setting up equations for $f_p$ and $f_u$  
+1 for correct $f_p$  
+1 for correct $f_u$  

+1 per blank (12 points total)  
1 sig fig, but answers should be exact
\[ u \text{ Cox} \frac{W}{L} = 1 \text{mA/V}^2 \quad |V_t| = 1 \text{V} \quad \lambda = 0.1 \text{ V}^{-1} \]

\text{a) } V_{BP} = 1.8 \text{V} \quad V_{DS} + \rho = V_{GP} - V_t = 3 - 1.8 - 1 = 200 \text{mV} \\
\quad \quad V_{DP} = 2.8 \text{V} \\
\quad \quad I_D = \frac{u \text{ Cox} }{2} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{PS}) \\
\quad \quad I_D = \frac{1}{2} \text{mA/V}^2 (0.2 \text{V})^2 \left(1 + \frac{0.1}{V} \times 0.2 \text{V}\right) = \frac{1}{2} \times 0.04 \text{mA}(1 + 0.02) \\
\quad \quad I_D = 20.4 \text{ uA} \\

\text{b) } \begin{array}{c}
\text{max} = 26 \text{ uA} \\
\text{min} = 20.4 \text{ uA}
\end{array} \\
\begin{array}{c}
2.8 \text{V} \\
3 \text{V} \\
V_{out}
\end{array}

\text{J\text{Lmax} = \frac{1}{2} mA/V^2 (0.2)^2 \left(1 + \frac{0.1 \times 3}{V}\right) = 26 \text{ uA}}

\text{c) } I_D = 20.4 \text{ uA} \quad V_o = 2.8 \text{V} \\
\quad \quad I_D = \frac{1}{2} \frac{mA}{V^2} (V_s - V_t)^2 \left(1 + 0.1 (2.8)\right) \quad V_i = 1.18 \text{V}
c) (cont.)

\[ V_{out} \]

\[ 0.2 \] \[ 2.8 \] \[ 3 \]

\[ I_{dp} \]

\[ I_{dn} \]

\[ +1 \] for equation to find \( V_i \)
\[ +1 \] for \( V_i \) within 0.1 V
\[ +1 \] for \( I_{dn} \) curve
\[ +1 \] for \( I_{dp} \) curve

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d) NMOS enters saturation at 0.2 V \( V_{out} \)

\[ I_{dp} \] at \( V_{out} = 0.2 \) V is:

\[
\frac{1}{2} \frac{mA}{V^2} (0.2)^2 \left( 1 + 0.1(2.8) \right) = 25.6 \mu A
\]

\[ 25.6 \mu A = \frac{1}{2} \frac{mA}{V^2} (V_i - V_t)^2 \left( 1 + 0.1(0.2) \right) \]

\[ V_i = 1.22 \ V \]

\[ +1 \] for equation to find \( V_i \)
\[ +1 \] for \( V_i \) within 0.1 V
\[ +1 \] for \( I_{dn} \) curve (including finding the current at \( V_{out} = 0.2 \) V)
\[ +1 \] for \( I_{dp} \) curve

---

e) \( V_o \)

\[ 3V \]

\[ (1.18V, 2.8V) \]

\[ (1.22V, 0.2V) \]

\[ 1.18 \] \[ 1.22 \] \[ 3V \]

\[ V_i \]

7 pts total

\[ +1 \] nmos off and pmos off regions
\[ +1 \] nmos triode (roughly quadratic)
\[ +1 \] pmos triode (roughly quadratic)
\[ +2 \] for the high gain start (x,y) points
\[ +2 \] for the high gain end (x,y) points
\[ A_{v0} = \frac{\Delta V_o}{\Delta V_{in}} = 2.6 \text{ V} \]

| \( A_v = -65 \text{ V/V} \) |

Output range is 0.2 V to 2.8 V (2.6 V swing)
Input range is 1.18 V to 1.22 V (40 mV)

9) \( A_v \) \( V_{o,pc} = 2.8 \text{ V} \)

\[ g_m = \frac{2I_0}{V_oV} = \frac{2 (20.4 \text{ uA})}{1.18 - 1} = 230 \text{ uS} \]

\[ r_o = \frac{1}{2} \frac{1}{\kappa I_D} = \frac{1}{2} \frac{1}{0.1 (20.4 \text{ uA})} = 245 \text{ kΩ} \]

Or \[ A_v = -g_m r_o = -\frac{1}{\lambda V_o V} = -\frac{1}{(0.1)(0.18)} = -55.6 \text{ V/V} \]

At \( V_{o,pc} = 0.2 \text{ V} \), \( I_D = 25.6 \text{ uA} \)

\[ A_v = -\frac{1}{\lambda V_{ov}} = -\frac{1}{0.1 (0.22)} = -45.5 \text{ V/V} \]

At \( V_{o,pc} = 1.5 \text{ V} \), \( I_D = \frac{1}{2} \text{ mA/V}^2 (0.2 \text{ V})^2 (1 + 0.1(1.5)) = 23 \text{ nA} \)

\[ A_v = -\frac{1}{\lambda V_{ov}} = \frac{1}{(0.1)(0.2)} = -50 \text{ V/V} \]

(NMOS \( V_{ov} \) must be 0.2 V)

In summary:

<table>
<thead>
<tr>
<th>( V_{o,pc} )</th>
<th>( A_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8 V</td>
<td>-55.6 V/V</td>
</tr>
<tr>
<td>1.5 V</td>
<td>-45.5 V/V</td>
</tr>
<tr>
<td>0.2 V</td>
<td>-50 V/V</td>
</tr>
</tbody>
</table>
HW3 grading rubric

1) 28 pts total
   1a) 2
   1b) 4
   1c) 2
   1d) 2
   1e) 9 (0.5 for each entry in the table)
   1f) 9, 1 for each plot

2) 8 pts total
   2a) 2 pts
   2b) 4 pts; 1 for right answer, 1 for some reasoning, 2 for error calc
   2c) 2 pts.

3) 4 pts, 2 for each frequency

4) 12 pts, 1 per blank

5) 33 pts total
   5a) 4 pts
   5b) 6 pts: 2 for plot, 2 each for min/max Idp
   5c) 4 pts: 2 for Vi ; 2 for plot
   5d) 4 pts: 2 for Vi ; 2 for plot
   5e) 7 pts: 1 for "nmos off" region, 1 for pmos triode, 1 each for the X and Y location of the beginning of high gain, 1 each for the X and Y location of the end of high gain, 1 for nmos triode. The triode regions should be vaguely quadratic, and the high gain region should be a straight line.
   5f) 2 pts
   5g) 6 pts