1. [35] In a simple spring/mass/damper MEMS device with a large $Q$, you have measured the important parameters as $K$, $M$, and $B$.
   
   a. The force on the mass due to a constant acceleration $a$ is
      
      
   b. The displacement of the mass due to a constant acceleration $a$ is
      
      
   c. The resonant, or natural, frequency is
      
      
   d. Using parts b and c, the displacement of the mass due to a constant acceleration $a$ in terms of only $a$ and $\omega_n$ is
      
      
   e. Using part d, a system with a natural frequency of $\omega_n = 1,000$ rad/s under an acceleration of $1g=9.8\text{m/s}^2$ has a displacement of (numerical answer in meters please)
      
      
   f. A system with a $Q$ of 100 and a static displacement of 1nm under a 1g field is shaken at its resonant frequency by an acceleration with a magnitude of 1g. What is the magnitude of the displacement?
      
      
   g. A system with a $Q$ of 100 and a static displacement of 1nm under a 1g field is shaken at 100 times its resonant frequency by an acceleration with a magnitude of 1g. What is the magnitude of the displacement?
2. [25] You have a comb-drive resonator running in air, and you apply an electrostatic force with constant magnitude but variable frequency and measure the resulting displacement. At very low frequency you find that the displacement is relatively constant at 10nm. As you increase the frequency of the applied force the displacement peaks at 500nm at 10kHz and then falls off rapidly above that frequency
   a. What is the Q of the device in air?

   

   b. You put the device in a vacuum chamber, and reduce the air pressure by a factor of 10, thereby reducing the air viscosity by a factor of 10. You make the same tests as above.
      i. Do you expect the damping coefficient B to increase, decrease, or stay about the same (be specific).
      ii. Do you expect the resonant frequency to increase, decrease, or stay about the same?
      iii. Do you expect the displacement at low frequency to increase, decrease, or stay about the same?
      iv. Do you expect the displacement at the resonant frequency to increase, decrease, or stay about the same?

3. [30] You have a second comb-drive resonator also running in air. It is identical to the first, with the exception that the width of the folded flexure beams has been increased from 2um to 4um. It is driven with the same magnitude force as above. I’m looking for numerical answers when possible, like “decrease by a factor of 5”
   a. How will the spring constant change?
   b. How will the resonant frequency change?
   c. How will the damping coefficient change?
   d. How will the Q change?
   e. Roughly what amplitude will you measure at very low frequency?
   f. Roughly what amplitude will you measure at the resonant frequency?
4. [15] Four silicon resistors are in a Wheatstone bridge with an excitation voltage of 1V. All of the resistors are nominally the same size. Assume that the silicon has a temperature coefficient of resistance of 0.1%/C, and a piezoresistive gauge factor of 100.
   a. If one resistor is 0.1% larger than the rest, what is the magnitude of the bridge output voltage?  
      \[ V_{out} = \] 
   b. One of the resistors is stretched, increasing its length by \( 10^{-6} \) (1 micro-strain). What is the percent change in resistance, \( \Delta R/R \)?  
      \[ \Delta R/R = \] 
   c. One of the resistors is 1 degree Celsius warmer than the rest. What is the percent change in its resistance?  
      \[ \Delta R/R = \] 

5. [20] In an SOI process, you have a silicon beam 1mm long, and 2um wide by 50um tall in cross-section at room temperature. Assume a thermal conductivity of \( \kappa = 100 \text{W/(m K)} \), a thermal expansion coefficient of \( 2.5 \times 10^{-6} / \text{K} \), and a Young's modulus of 150GPa. [for the math-without-silicon-challenged: \( A = 10^{-10} \text{m}^2 \), \( EI = 5 \times 10^{-12} \text{Nm}^2 \)]
   a. If you apply a temperature difference of 100 degrees across the two opposite cross-sectional faces (1mm apart), how much heat flows down the length of the beam?  
      \[ Q_{dot} = \] 
   b. If you uniformly heat the beam by 400C and it is allowed to expand, what is the change in length?  
      \[ \Delta L = \] 
   c. If the ends of the beam are clamped in place and the beam is heated until it is under a strain of \( 10^{-6} \), what force will it exert on the clamps?  
      \[ F = \] 
   d. If the ends of the beam are clamped in place, what is the buckling load?  
      \[ F = \] 

6. [15] You have an air-gap parallel plate capacitor with a potential \( V_{DC} \) on one plate, and \( V_{AC}\sin(\omega t) \) on the other. What is \( F(t) \), the time-varying force of attraction between the two plates? Plate area=\( A \), gap=\( g \). Be specific about the frequency content of the force.
7. [26] True/False (circle one) 2pts each
   a. T / F Silicon is stiffer (larger Young’s modulus) than aluminum
   b. T / F The first 10 mode shapes of the beam in hw7 looked roughly the same.
   c. T / F In Coventor, to “mesh” something means to create a new MEMS process.
   d. T / F Aluminum makes a good mask for KOH etching
   e. T / F All quadratic energy storage modes in a MEMS device have $K_B T/2$ of average thermal noise energy in them.
   f. T / F It’s OK to deposit SiO2 onto aluminum by LPCVD with an LTO process.
   g. T / F It’s OK to deposit SiO2 onto photoresist by LPCVD with an LTO process.
   h. T / F Sputtering systems can only deposit materials with a low melting temperature
   i. T / F Water “beads up” on hydrophobic surfaces
   j. T / F Chemical etches often have high material selectivity
   k. T / F Plasma etchers use electrons to physically knock atoms off the wafer
   l. T / F Metal deposition by evaporation is a conformal process.
   m. T / F Fluorine atoms etch silicon anisotropically.

8. [35] Short answer – at most one sentence needed. Sentence fragments ok.
   a. List three different processes or design strategies to minimize stiction during release or operation of a MEMS device, and how they help
      i. 
      ii. 
      iii. 
   
b. Why do metal evaporation systems run at high vacuum?

c. What causes stringers in surface micromachining?

d. What are the two phases of the Bosch DRIE process, and what is etched or deposited in each phase?
   i. 
   ii. 

9. [] Use a piece of scratch paper to build a structure strong enough to support your weight 3m above your desk. (just kidding)
10. [20] In the following figures, assume that a thin film of thickness 0.5um has been patterned with a 0.5um line and space on three different wafers. The cross section of two lines is shown below. Draw the expected deposition contour for three different depositions: a 0.1um conformal silicon nitride, a 1um thick spin-cast photoresist, and a “hemispherical source” sputtering of 0.2um aluminum.

![LPCVD 0.1um silicon nitride](image1)

Spin-cast 1um photoresist

Sputter 0.2um aluminum

11. [20] In the following figures, a 1 micron thick SiO2 layer has been patterned to expose bare [100] silicon on three different wafers. The aperture is 10um square. Draw the cross section for each of the following three etches: 10um deep etch in KOH, 10um deep etch in DRIE, 10um deep etch in XeF2.

![10um KOH](image2)

10um DRIE

10um XeF2
12. [40] An accelerometer has a proof mass $M$ supported by a spring $K$ with one primary degree of freedom in direction $x$.

a. Write an equation relating, $x_n$, the average displacement of the spring due to thermal noise, to the temperature $T$.

b. If $K=40\,\text{N/m}$ and $k_B T=4\times10^{-21}$, what is the average displacement of the spring due to thermal noise?

\[ x_n = \]

c. Write an equation relating the average velocity of the proof mass in direction $x$ to the temperature.

d. Write an equation for $P_n$, the thermal noise power, in a bandwidth $\Delta F$ due to a damper $B$

e. Write an equation for $F_n$, the thermal noise force, in a bandwidth $\Delta F$ due to a damper $B$

f. If the thermal noise force in a 1Hz bandwidth is 0.4pN, and $K=40\,\text{N/m}$, and $Q=100$

i. What is the displacement of the proof mass due to thermal noise in a 1Hz bandwidth at low frequency?

\[ x_n = \]

ii. What is the displacement of the proof mass due to thermal noise in a 100Hz bandwidth at low frequency?

\[ x_n = \]

iii. What is the displacement of the proof mass due to thermal noise in a 1Hz bandwidth at the resonant frequency?

\[ x_n = \]

13. [5] You have a pressure sensor with a linear relationship between the measured pressure $P$ and the output voltage $V$ of 1V/atm. You measure the noise voltage on the output of the pressure sensor to be approximately 10uV. What is the noise-equivalent pressure, $P_{NE}$?

\[ P_{NE} = \]