1. In an SOI process with a 20um thick device layer you have a very thin piezoresistive layer on the top surface. If you make a beam that is 4um wide and 1mm long, and apply a force perpendicular to the wafer surface.
   a. Find the spring constant of the beam
      1 pt. for approximately right answer

      E= 169GPa. As stated in the problem, a=2e-5m, b=4e-6m, L=1e-3m.

      \[ k = \frac{Ea^3b}{4L^3} \]

      \[ k = 1.35 \frac{N}{m} \]

      If you swapped a and b and got 0.054N/m, then lose that 1 point here, but give yourself full credit below as long as you were consistent.

   b. Find the strain at the surface of the beam as a function of force
      1 pt. for correct equation

      \[ \epsilon(x, z) = \frac{z}{\rho(x)} \]

      \[ \epsilon \left( x = 0, z = \frac{a}{2} \right) = \frac{a}{2} \frac{\rho(x)}{\rho(x)} = \frac{a}{2} \]

      \[ \epsilon \left( x = 0, z = \frac{a}{2} \right) = \frac{a}{2} \frac{M(x)}{EI} = \frac{a}{2} \frac{FL}{EI} \]

      \[ \epsilon \left( x = 0, z = \frac{a}{2} \right) = \frac{6FL}{Ea^2b} \]

      \[ \epsilon = 22[1/N] F \]

      So a 1 μN force gives 22 microstrain.

   c. Find the strain at the surface of the beam as a function of deflection
      1 pt. for correct equation

      \[ \epsilon \left( x = 0, z = \frac{a}{2} \right) = \frac{a}{2} \frac{FL}{EI} \]
Knowing $F = ky$, plug it into equation above to obtain:

$$\varepsilon \left( x = 0, z = \frac{a}{2} \right) = \frac{3}{2} \frac{ay}{L^2}$$

$$\varepsilon = 30 \left[ \frac{1}{m} \right] y$$

So 1um deflection gives 30 microstrain.

You make a 1kΩ strain gauge at the base of the beam, and put it in a Wheatstone bridge with a 2V excitation voltage. The resistor has a gauge factor of +30, and a TCR of +0.1%. You have an instrumentation amplifier with a noise-limited resolution of 1µV.

d. Find the bridge output voltage as a function of strain
   1 pt. for correct equation

In this case we are disregarding any changes in temperature (they are non-existent). $G$ is the gauge factor, and $V_x$ is the excitation voltage:

$$V_{out} = \frac{G \varepsilon}{4} V_x$$

$$V_{out} = 15 \left[ V \right] \varepsilon .$$

So 1 microstrain in the resistor causes a 15µV change in the output voltage.

Note that we don’t know the sign, since we don’t know where the resistor is in relation to an arbitrarily-defined positive output voltage in the bridge.

e. Find the minimum detectable force
   1 pt. for approximately right answer

Plugging in the equation for strain in part b into the equation in part d, we obtain:

$$V_{out} = \frac{G}{2} \frac{a}{E} \frac{FL}{EI} V_x$$

The minimum output voltage is limited by the noise of the amplifier, and is given to be 1µV. You can plug this into the equation above and solve for the minimum detectable force:

$$F_{min} = 4 \frac{EIV_{out,min}}{G} \frac{a}{L} V_x$$
\[ F_{\text{min}} = 3nN \]

f. **Find the minimum detectable deflection**  
1 pt. for approximately right answer

Using the equation \( F = ky \), and the known spring constant and force from part d, we can solve for the minimum detectable displacement.

Also, you can plug the equation for strain in part c into the equation in part d, and solve for the displacement.

\[ y_{\text{min}} = 2.22\,\text{nm} \]

g. **Find the temperature change that gives a noise-equivalent bridge output voltage**  
1 pt. for approximately right answer

Assuming no strain, the output voltage equation for the Wheatstone bridge becomes:

\[ V_{\text{out}} = \frac{\alpha \Delta T}{4} V_x \]

Solving for \( \Delta T_{\text{min}} \) in Kelvin:

\[ \Delta T_{\text{min}} = \frac{4V_{\text{out, min}}}{\alpha V_x} \]

\[ \Delta T_{\text{min}} = 2\,\text{mK} \]

h. **Find the power dissipation in the piezoresistor**  
1 pt. for approximately right answer

The current in each branch is approximately (considering an ideal op amp will have infinite impedance at the input):

\[ \frac{2V}{2R_0} = 1mA \]

The power dissipated in each resistor is

\[ I^2 R \approx 1mW \]

\[ P = 1mW \]

This is an approximation of the power dissipated by the piezoresistor.
i. Find the thermal resistance for which the power dissipated in the resistor generates a temperature change that gives a noise-equivalent bridge output voltage

1 pt. for approximately right answer

The power in the electrical domain is equal to the heat transfer in the temperature domain.

\[ \dot{Q} = \Delta T \frac{1}{R_{th}} \]

Where in this case \( \dot{Q} \) is the 1mW dissipated in the piezoresistor from part h, and \( \Delta T \) is the 2mK temperature change found in part g.

\[ R_{th} = 2 \frac{K}{W} \]

j. Find the thermal resistance of the silicon beam as a function of its length

1 pt. for correct equation

\( \mathcal{K} \) is the thermal conductivity of the material. For single-crystal silicon, its value is 150 [W/mK].

\[ R_{th}(L) = \frac{L}{\mathcal{K}A} \]

\[ R_{th} = 8e7 [K/mW] L \]

So a 1 µm long beam has a thermal resistance of 80 [K/W].

k. How long can you make the piezoresistor before you have to worry about temperature effects?

1 pt. for approximately right answer

Using the value from part i, find a value for L:

\[ L = R_{th}(L)\mathcal{K}A \]

Using \( \mathcal{K} = 150 \) for single crystal silicon, a thickness of 20µm and a width of 4µm for the area:

\[ L = 24nm \]