Homework Assignment #5
Due by online submission Friday 9/30/2016 (Saturday 9am)

1. For a Tang-style comb drive resonator on POLY1 in polyMUMPS with \( K=10 \text{N/m}, m=10^{-11} \text{kg}, \)
\( b=10^{-7}, \) and 100 electrostatic gaps per side
   a. Calculate the resonant frequency and the quality factor
   b. Calculate the DC deflection with 1.5, 15, and 150V applied
   c. With a 1.5V sine wave at 1 Hz applied to one comb drive input while the structure is
      biased at -150V,
         i. Calculate the magnitude of the DC, 1 Hz, and 2 Hz forces
         ii. Calculate the deflection (zero to peak) due to those forces
   d. With the 1.5V sine wave input at the resonant frequency of the structure, and the same
      DC bias as above
      i. Calculate the deflection at \( \omega_n \) and \( 2\omega_n \).
      ii. If you cut the input frequency in half, calculate the response at \( 1/2 \omega_n \) and \( \omega_n \).

2. If viscosity is proportional to air pressure, and using the resonator in the previous problem driven
   at its natural frequency
   a. how much does the amplitude of vibration of a comb drive resonator change due to
      normal daily changes in air pressure (assuming constant drive voltage, etc.)?
   b. if you vary the pressure from STP conditions to 100 Pa, how much would the amplitude
      increase?
   c. if that amplitude increase is too much, and you decide to keep the amplitude of vibration
      constant by changing the amplitude of the AC drive voltage, how much does the voltage
      change from STP to 100 Pa?
   d. if the resonator itself has an intrinsic Q of 10,000, what is the minimum detectable
      pressure?

3. You have made an array of Tang-style resonators in an SOI process, in which you vary the width
   of the support beams from 2 to 5.5um (drawn). You know that the fabrication process always
   has a slight under-etch or over-etch, and the fabricated width, \( a_f \), of all beams is narrower or
   wider than the drawn width, \( a_d \), by a constant factor, \( \delta a \), so \( a_f = a_d + \delta a \).
   a. Write an expression for the resonant frequency as a function of the constant offset in
      beam width.
   b. Measuring resonant frequency and plotting vs. drawn beam width is easy, but it’s tough
      to figure out the offset from that nonlinear curve. Figure out how to get a plot that is
      linear in \( a_f \), so that the x offset will tell you \( \delta a \), and the slope will tell you something
      about the stiffness to density ratio of the material.
   c. You have tested the devices measure their resonant frequency. The design and process
      information and test results are below. Use this data to estimate the over-etch or under-
      etch in the process, and the ratio between the Young’s modulus and the density.

<table>
<thead>
<tr>
<th>Young's Modulus</th>
<th>1.69E+11</th>
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<tbody>
<tr>
<td>Density (kg/m^3)</td>
<td>2300</td>
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<tr>
<td>Spring Length (m)</td>
<td>3.00E-04</td>
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<tr>
<td>Dev. Thickness (m)</td>
<td>4.00E-05</td>
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<tr>
<td>Shuttle Area (m^2)</td>
<td>3.62E-08</td>
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<tr>
<td>Shuttle Mass (kg)</td>
<td>3.33E-09</td>
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</tbody>
</table>

Measured
4. [247A] Find the temperature coefficient for viscosity and silicon Young’s modulus, and estimate how Q and resonant frequency change with temperature. Can you make a temperature sensor if you have a good clock? If you have a clock with 10 ppm temperature stability, how good will your temperature sensor be?