NO CALCULATORS, CELL PHONES, or other electronics allowed. Show your work, and put final answers in the boxes provided. **Use proper units in all answers.**

1. [6] In the figure below, the structure on the left is the pattern on a mask. The mask is used for a Deep Reactive Ion Etch (DRIE) into a Silicon-on-insulator (SOI) wafer with a top-layer thickness of 20μm, and an oxide thickness of 4μm. Two copies of a cross-section of the device are shown after the DRIE etch and subsequent photoresist removal. The device is then dropped into a hydrofluoric (HF) acid solution with an SiO₂ etch rate of 1μm/minute.

   a. Carefully draw the process cross-section after 1 minute in HF.

   b. Carefully draw the cross-section after 4 minutes in HF.

   c. On a top-down view of the structures, carefully sketch where there would still be oxide in contact with the SOI layer after a 1 minute etch.

   ![Diagram of mask and cross-sections](image)

   part c, 1 minute etch

2. [3] With the mask above, you run two different variants of the oxide etch process on two different wafers:

   a. reactive ion etch (RIE) with vertical sidewalls

   b. reactive ion etch (RIE) with vertical sidewalls, followed by 49% HF etch with 1μm etch distance

   Draw the cross section after each etch.

   ![Diagram of cross-sections](image)

   part a, 1 minute etch

   part b, 4 minute etch

   part a

   part b
3. [6] You have two conductive parallel plates that are 10um square (10x10um2) separated by a gap of 1um. One plate is grounded, the other is biased at 15V. A cross-section is shown above.
   a. What is the approximate magnitude of the force between them? You may ignore fringing fields.
      \[ F = \frac{1}{2} \varepsilon_0 V \frac{A}{g^2} (1N) \text{ (10)} \]
      +1 pt. # 1 pt. unit
      \[ F = 100nN \]
   b. If the potential on the biased plate is switched from 15V to -15V, how does the force change? (increase by ..., decrease by ..., change sign, stay the same, ...)
      +1 pt.
      same
   c. If the potential on the biased plate is switched from 15V to 150 V, how does the force change?
      +1 pt.
      Increase by 100
   d. At 15V, if the gap between the plates is decreased to 0.1um, how does the force change?
      +1 pt.
      Increase by 100

4. [5] You have three conductive parallel plates that are 10um square (10x10um2) separated by gaps of 1um. The middle plate is offset from the other two by 5um. The middle plate is biased at 15V. A cross-section is shown above.
   a. What is the approximate vertical force (y axis) on the middle plate?
      +1 pt. # 1 pt. unit
      \[ F_y = 0N \]
   b. What is the approximate horizontal force (x axis) on the middle plate?
      \[ F = 2 \left( \frac{1}{2} \varepsilon_0 V^2 \right) \frac{g}{2} = 2(1n) \left( \frac{10mm}{1um} \right) = 20nN \]
      +1 pt. # 1 pt. unit
      \[ F_x = 20nN \]
   c. If the middle plate is offset further, to a total of 9um offset (1um overlap), how does the horizontal force change? (increase by ..., decrease by ..., change sign, stay the same, ...)
      +1 pt.
      same
5. [8] In the polyMUMPS process, you draw a 2um wide, 200um long cantilevered beam (anchored on one end, free on the other) on layer POLY1. Assume E=150 GPa
   a. If you apply an axial force of 1uN at the tip, ignoring stress concentrations
      i. what is the stress near the base of the beam?
      \[ \sigma = \frac{F}{A} = \frac{1 \times 10^{-6} \text{N}}{4 \times 10^{-4} \text{m}^2} = \frac{1}{4} \times 10^2 \text{ MPa} = 250 \text{ kPa} \]
      ii. what is the strain near the base of the beam?
      \[ \epsilon = \frac{\sigma}{E} = \frac{2.5 \times 10^8 \text{ Pa}}{1.5 \times 10^{12} \text{ Pa}} = \frac{5}{3} \times 10^{-6} = 1.7 \text{ um} \]
   b. If you apply a transverse force of 1uN at the tip
      i. What is the moment near the base of the beam?
      \[ 12 = (2 \times 10^6 \text{ N} \cdot \text{m}) \]
      ii. What is the maximum strain near the base of the beam?
      \[ \epsilon = \frac{2 \times 10^{-6} \text{ m}}{12} \]
      \[ = \frac{16 \times 10^{-4} \text{ m}}{4\times 10^{-14} \text{ m}^4} \]
      \[ = 4 \times 10^{-13} \text{ m} \]

6. [10] In a Tang-style resonator, you have set up your biases so that there is 1uN at DC, 0.1uN at the AC supply frequency \( \omega \), and 0.01uN at twice the AC supply frequency. The spring constant in your device is 1 N/m, and the resonant frequency \( \omega_r \) is approximately 10 kHz. The Q of the resonator is roughly 50.
   a. When \( \omega = 1 \text{ Hz} \), what is the amplitude of the displacement at DC, \( \omega \), and 2 \( \omega \)?
      \[ x_{DC} = 1 \text{ um} \]
      \[ x_{\omega} = 0.1 \text{ um} \]
      \[ x_{2\omega} = 0.01 \text{ um} \]
   b. When \( \omega = \omega_r / 2 \), what is the amplitude of the displacement at \( \omega \), and 2\( \omega \)?
      \[ \omega = \frac{\omega_r}{2} \]
      \[ \omega_n = 50(0.01 \text{ m}) \]
      \[ x_{\omega} = 0.13 \text{ um} \]
      \[ x_{2\omega} = 0.05 \text{ um} \]
   c. When \( \omega = \omega_n \), what is the amplitude of the displacement at \( \omega \)?
      \[ 50(0.01 \text{ m}) \]
      \[ x_{\omega} = 0 \]
   d. If the resonator is driven as in part c, how does the displacement change if the ambient pressure drops by a factor of 2? (increase by ..., decrease by ..., about the same, ...)
      \[ \text{increase 2x} \]

7. [6] You have made a cantilevered beam with a single strain gauge at the base. You use that strain gauge in a Wheatstone bridge made with three other resistors, nominally the same value as the strain gauge. Your strain gauge has a gauge factor of 100, and a temperature coefficient of resistance of 2%/K. The bridge excitation is 1V.
   a. What is the magnitude of the bridge output due to an applied strain of 1 ppm (part per million)?
      \[ \frac{V_o}{4} = 1V \frac{(100)(10^6)}{4} = 25 \mu V \]
      \[ V_{out} = 25 \mu V \]
   b. What is the bridge output due to a temperature change in the strain gauge of 1 K?
      \[ \frac{V_o}{4} = 1V \frac{(0.02)(1)}{4} = 5 \mu V \]
      \[ V_{out} = 5 \mu V \]
   c. If your electronics can detect signals down to 1\( \mu V \), what is the minimum detectable strain?
      \[ \frac{1 \text{ ppm}}{25} = 40 \text{ nm} \]