

Problem Set 2

EE221a: Linear Systems Theory

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- Exercise 3, pg. 28 of the xeroxed "Notes for a Second Course in Linear Systems", by Desoer. The pagination refers to the original page number in the text. The problem requires you to use row and column operations to change bases in the domain and codomain of a given matrix $A \in \mathbb{F}^{m \times n}$ to express it as a "diagonal" matrix $D \in \mathbb{F}^{m \times n}$ with 1s and 0s on the main "diagonal".
- Let $A \in \mathbb{C}^{n \times n}$, $u, v \in \mathbb{C}^n$. Suppose that you know A^{-1} exists. Show that

$$(A + uv^*)^{-1} = A^{-1} - \frac{A^{-1}uv^*A^{-1}}{1 + v^*A^{-1}u}$$

- Let $A \in \mathbb{C}^{m \times n}$, $B \in \mathbb{C}^{n \times q}$, $C \in \mathbb{C}^{m \times n}$, $D \in \mathbb{C}^{n \times q}$ be given. When are each of the matrix equations

$$AX = C \quad XB = D$$

solvable for $X \in \mathbb{C}^{n \times n}$. When are the solutions unique?

- From text by Callier and Desoer, pg.420**

Let A be a linear map of (U, F) into itself with dimension n . (i) Suppose that $A^n = -\alpha_1 A^{n-1} - \alpha_2 A^{n-2} - \dots - \alpha_n I$, where I is the identity map: $U \rightarrow U$, and $\alpha_i \in F$. Suppose that b is an A generator of U , i.e. $(b, Ab, \dots, A^{n-1}b)$ is a basis of U . Show that with respect to this basis, the vector b and the linear map A are represented by

$$b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\alpha_n \\ 1 & 0 & \cdots & 0 & -\alpha_{n-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 1 & -\alpha_1 \end{bmatrix}$$

- From text by Callier and Desoer, pg.421**

Consider the same set up as in the previous problem and assume that for some $\lambda \in F$ and a different basis $(v_i)_{i=1}^n$ we have

$$Av_1 = \lambda v_1$$

and

$$Av_k = \lambda v_k + v_{k+1} \quad k = 2, \dots, n$$

Obtain a representation of A with respect to this basis.

Linear Algebra References for Continued Linear Algebra Review

- Ben Noble and James Daniel, "Applied Linear Algebra," Prentice Hall, 3rd Edition, 1988.

2. Gilbert Strang, "Introduction to Linear Algebra", Wellesley Cambridge Press, 1993.
3. Nathan Jacobson, "Basic Algebra I", Freeman, 1974.
4. J. H. Wilkinson and C. Reinsch, "Linear Algebra", Springer Verlag, 1971.
5. G. H. Golub and C. van Loan, "Matrix Computations", Johns Hopkins Press, 3rd edition, 1996
6. Gantmakher, F. R. "Applications of the Theory of Matrices," Wiley Interscience Publishers, 1959.