University of California, Berkeley
Department of EECS

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Assigned: February 1

Spring 2007
Due: **February 15**

EE222: Homework Assignment 2

1. Draw the phase portrait of the reaction-diffusion system

$$\begin{aligned}\dot{x}_1 &= 2(x_2 - x_1) + x_1(1 - x_1^2)\\ \dot{x}_2 &= -2(x_2 - x_1) + x_2(1 - x_2^2).\end{aligned}$$

List the equilibria and their types. Does the system have limit cycles?

2. Use all the techniques you learned so far to make the phase portraits shown in Figure **??** plausible. You are only allowed to change the directions of some of the orbits or add new ones, but you are not allowed to delete any existing orbits.

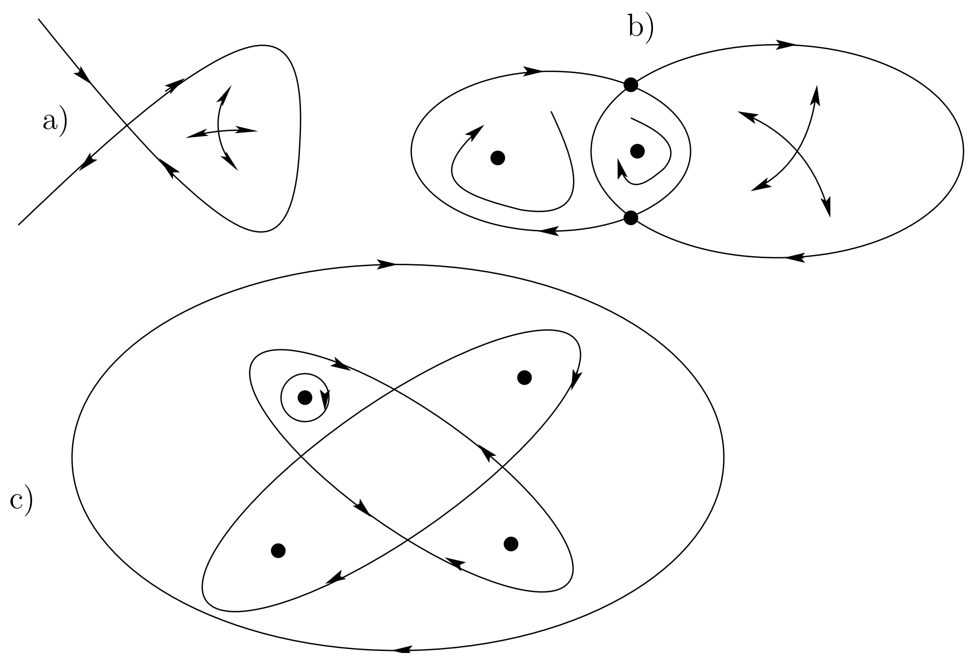

Figure 1: Problem 2.

3. Draw bifurcation diagrams of the following 1-dimensional systems as μ varies:
(a) $\dot{x} = \mu^2 x - x^3$

(b) $\dot{x} = \mu^2 \alpha x + 2\mu x^3 - x^5$, for different α.

4. Let f be a smooth vector field on the annulus

$$A = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4\}.$$

Assume f points inward along the boundary of A, and that for every $0 \leq \alpha \leq 2\pi$, the radial segment (in polar coordinates)

$$S_\alpha = \{(r, \theta) : 1 \leq r \leq 2, \quad \theta = \alpha\}$$

is a **local cross section** in the sense defined in class; that is, at every point $p \in S_\alpha$, the angle between $f(p)$ and S_α is not zero.

(a) Let $p \in S_0$ be an arbitrary point. Show that the orbit of f starting at p returns to S_0 after some positive time.

(b) Show that every continuous function $\phi : [1, 2] \to [1, 2]$ has a fixed point.

(c) For $p \in S_0$, let $\psi(p)$ be the point of first return to S_0 of the orbit starting at p. Show that $\psi : S_0 \to S_0$ has a fixed point.

(d) Show that there exists a closed orbit of f in A.

5. Consider the following family of differential equations, parametrized by real numbers μ_1, μ_2:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \mu_1 x + \mu_2 y + x^3 - x^2 y. \end{aligned}$$

(a) What does the flow near $(0, 0)$ look like when $\mu_1 = \mu_2 = 0$?
(b) What bifurcations occur at $\mu_1 = 0$ and at $\mu_2 = 0$ (for $\mu_1 < 0$)?
(c) Use Bendixson's theorem and index theory to rule out parameter regions where there are no periodic orbits.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a diffeomorphism; in other words, f is invertible, and both f and f^{-1} are smooth maps. Let $p_0, p_1, \ldots, p_{N-1}$ be a period N cycle of f; that is, $p_k = f^k(p_0)$ for $1 \leq k \leq N-1$ and $f^N(p_0) = p_0$. Prove that the matrices

$$Df^N(p_k)$$

have the same spectrum for $k = 0, \ldots, N-1$.