Kinematics and Control of Multifingered Hands with Rolling Contact

ARLENE B. A. COLE, STUDENT MEMBER, IEEE, JOHN E. HAUSER, STUDENT MEMBER, IEEE, AND S. SHANKAR SASTRY, MEMBER, IEEE

Abstract—In this paper, we derive the kinematics of rolling contact for two surfaces of arbitrary shape rolling on each other. Applying these kinematic equations to a multifingered hand manipulating some object of arbitrary shape in three dimensions, a scheme is presented for the control of such a hand, which is in fact a generalization of the computed torque method of control of robot manipulators. In implementing the control, we require that all applied forces lie within the friction cone of the object so that sliding does not occur. The theory has been validated by dynamic graphical simulations of the resulting closed-loop system for several examples.

I. INTRODUCTION

An important feature of multifingered hands is their ability to perform fine motion manipulation, especially when the manipulator operates in a crowded environment. Most current control schemes for multifingered hands, for example [4], assume the contact type between the fingertip and the object to be of the point contact with friction type.

In this work, we consider the manipulation of objects of arbitrary shape by multifingered hands, where the contact type between the object and the fingers is a rolling contact, i.e., the fingertip rolls without slipping on the surface of the object. The kinematics, prehensibility,1 dynamics, control, and control of these systems are developed in this paper.

Recent work on rolling contacts is the work of Kerr [1], Montana [2], and Cai and Roth [3]. Kerr discusses how to compute the movement of the fingers in order to produce a given displacement of the object. Kinematic equations are derived from the constraint that the fingertip and object velocities are equal at the point of contact. Control of such a hand is not considered. Montana [2] studies the kinematics of contact from a geometric point of view. He does not, however, study the effects of the kinematics of a finger attached to the fingertip. Cai and Roth [3] study the roll-slide motions between two curved surfaces under planar motion. The kinematic equations for the contact point evolution are derived. To our knowledge, there has been no previous work on the control of multifingered hands with rolling contacts. Related work in multifingered hands is quite plentiful (see, for instance, Cutkosky [11], Mason and Salisbury [12], and Mishra et al. [13]).

A brief outline of the paper is as follows. In Section II we derive the kinematics of rolling in \( \mathbb{R}^3 \) using velocity constraints and normal constraints between the surfaces; the development closely parallels that of [1]. In Section III we derive relationships between the joint torques and velocities of the fingers and the net force and velocity of the body being manipulated. Section IV gives the control scheme along with a proof of its convergence. Section V presents dynamic simulation results of the application of our control law in the planar case, to a two-fingered hand with elliptical fingertips; and also in three dimensions, to a three-fingered hand with ellipsoidal fingertips manipulating an ellipsoidal object.

II. THE KINEMATICS OF ROLLING

When one surface rolls on top of another, the trajectory of the contact point on each surface depends in an essential way on the geometry of the surfaces. In this section, we derive the differential equations that specify the evolution of the point of contact. Since our eventual goal is to apply the theory to the manipulation of objects by multijointed fingers with rolling contacts, we will refer to one of the bodies in contact as the object and the other as the finger.

Consider a finger in contact with an object as shown in Fig. 1. Denote by \( C_0 \) a coordinate frame attached to the center of mass of the object, and by \( C_f \) one attached to the finger. Parameterize the object and the fingertip surface (locally) in \( C_0 \), and \( C_f \), as \( c_0(\xi_0) \) and \( c_f(\xi_f) \), respectively. Note that \( \xi_0, \xi_f \in \mathbb{R}^3 \) if the surfaces in question are in \( \mathbb{R}^3 \). We will also be interested in the manipulation of objects in the plane (\( \mathbb{R}^2 \)), in which case \( \xi_0, \xi_f \in \mathbb{R}^2 \). Further, let \( C_b \) be an inertial base frame. Define \( x_o, x_f \in \mathbb{R}^2 \) to be the positions of the origins of \( C_0 \), \( C_f \) in the base frame, and \( R_o, R_f \in \mathcal{SO}(3) \) (the group of \( 3 \times 3 \) orthogonal matrices with determinant +1) to be the rotation matrices giving the orientations of \( C_0, C_f \) in the base frame \( C_b \), respectively. It follows from elementary considerations that a point on the object with coordinate \( c_0(\xi_0) \) in the object frame has (base frame) coordinates given by

\[
x_o + R_o c_0(\xi_0).
\]

(2.1)

Frames \( C_0 \) and \( C_f \) may both move relative to the base frame \( C_b \) so that \( x_o, x_f, R_o, \) and \( R_f \) are all functions of time. The velocity of the origin of frame \( C_0 \) has a translational component \( v_o \in \mathbb{R}^3 \) given by

\[
v_o(t) = \dot{x}_o(t)
\]

(2.2)
and a rotational component \( \omega_r = (\omega_r, \omega_r, \omega_r)^T \in \mathbb{R}^3 \) such that
\[
R_\omega = (\omega_r \times) R_\omega = \begin{bmatrix}
0 & -\omega_r & 0 \\
\omega_r & 0 & -\omega_r \\
0 & \omega_r & 0
\end{bmatrix} R_\omega. \tag{2.3}
\]

Note that the matrix \( R_\omega R_\omega^T \) is skew-symmetric because \( R_x(t) \) is orthogonal. The matrix in (2.3) is referred to as \( (\omega_r \times) \) since its action on a vector \( x \in \mathbb{R}^3 \) is precisely \( \omega_r \times x \). We may also define \( \nu_r \) and \( \omega_r \) in a similar fashion, to be the translational and rotational components of velocity for the fingertip.

The act of one surface rolling without slipping on top of another yields three constraints on certain parameters relating the two bodies: the position of the point of contact, the velocity at the point of contact, and the surface normals at the point of contact. We use these constraints to determine the evolution of the contact point.

First, since we assume that there is no slipping at the point of contact, the velocity at the point of contact on the object must equal the velocity at the point of contact on the fingertip (with reference to the base frame), i.e.,
\[
\nu_o + \omega_r \times R_x \nu_o = \nu_f + \omega_f \times R_f \nu_f. \tag{2.4}
\]
Equation (2.4) may be rewritten as
\[
U_o \begin{bmatrix} \nu_o \\ \omega_o \end{bmatrix} = U_f \begin{bmatrix} \nu_f \\ \omega_f \end{bmatrix} \tag{2.5}
\]
where
\[
U_o = [I - \left( R_x \nu_o \right) \times], \quad U_f = [I - \left( R_f \nu_f \right) \times].
\]
Note that \( U_o \) maps the velocity of the object to the velocity at the point of contact, Equation (2.5) equates the two different expressions for the velocity of the point of contact between the object and the fingertip.

Next, since the fingertip and object stay in contact, we may express the position of the point of contact in two ways, i.e.,
\[
x_o(t) + R_x(t) c_o(t) = x_f(t) + R_f(t) c_f(t). \tag{2.6}
\]
Differentiating (2.6) yields [using (2.3)]
\[
\nu_o + \omega_r \times R_x \nu_o + R_x \omega_r \times c_o = \nu_f + \omega_f \times R_f \nu_f + R_f \omega_f \times c_f. \tag{2.7}
\]
Subtracting (2.4) from (2.7) yields
\[
R_x \omega_r \times c_o = R_f \omega_f \times c_f, \tag{2.8}
\]
To show the dependence of \( c_o \) and \( c_f \) on the surface parameters \( \xi_o \) and \( \xi_f \), respectively, (2.8) may be rewritten as
\[
R_x \frac{\partial c_o}{\partial \xi_o} = R_f \frac{\partial c_f}{\partial \xi_f}. \tag{2.9}
\]
In \( \mathbb{R}^3 \), (2.9) represents three equations in the four unknowns \( \xi_o, \xi_f \). Since \( \partial c_o / \partial \xi_o, \xi_f \) is the tangent to the curve \( c_o(t) \) (expressed in object coordinates), (2.8) and (2.9) provide a constraint on the tangent vectors of the contact curves on the fingertip and object.

Finally, since the two surfaces touch at the point of contact, they are on opposite sides of a common tangent plane and thus must have equal and opposite outward unit normal vectors at the point of contact. Thus, with \( \hat{n}_o \) and \( \hat{n}_f \) being the outward unit normal vectors to the surface of the object and fingertip (in their respective frames), at the point of contact, we have
\[
R_x \hat{n}_o = -R_f \hat{n}_f, \tag{2.10}
\]

Differentiating (2.10) yields [again using (2.3)]
\[
\omega_r \times R_x \hat{n}_o + R_x \frac{\partial \hat{n}_o}{\partial \xi_o} \xi_o + \omega_f \times R_f \hat{n}_f + R_f \frac{\partial \hat{n}_f}{\partial \xi_f} \xi_f = 0. \tag{2.11}
\]

Note that the rate of change of the normal vectors (in the base frame) consists of a term due to body rotation (e.g., \( \omega_r \times R_x \hat{n}_o \)) and a term due to surface curvature (e.g., \( R_x \partial \hat{n}_o / \partial \xi_o \)). Again, as in (2.9), (2.11) is a set of three equations in four unknowns \( \xi_o, \xi_f \). Equations (2.6)-(2.11) above correspond to \[1. \) eq. (6.8)-(6.13).

Combining (2.9) and (2.11) yields
\[
\begin{align*}
\begin{bmatrix}
R_x \frac{\partial c_o}{\partial \xi_o} - R_f \frac{\partial c_f}{\partial \xi_f} \\
R_x \frac{\partial \hat{n}_o}{\partial \xi_o} - R_f \frac{\partial \hat{n}_f}{\partial \xi_f}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
(\nu_o, \omega_o) \times & (\nu_f, \omega_f) \times
\end{bmatrix}
\end{align*}
\tag{2.12}
\]

Equations (2.12) represent six equations which may be solved for \( \xi_o \) and \( \xi_f \) given \( \omega_o \) and \( \omega_f \). These equations are consistent, however, and they only provide four independent constraints. In particular, note that (2.10) provides two constraints since the normals are of unit length. Then, since any tangent vector at the contact point must lie in the common tangent plane, (2.9) only provides two more constraints. Thus, the six equations in (2.12) provide the four constraints needed to solve for \( \xi_o \) and \( \xi_f \).

These equations can be further simplified in the planar case. The interested reader may refer to [9], where the dependence of the contact point evolution on the curvature of the surfaces at the contact point is explicitly shown.

III. GRASPING AND MANIPULABILITY

In Section II, we dealt with the evolution of the point of contact for one finger rolling on an object. We will now consider a system of \( m \) fingers contacting an object, each finger with a rolling contact, and we will study the grasping and manipulability of such a hand-object system. Let the \( m \) finger frames be \( C_{f_1}, C_{f_2}, \ldots, C_{f_m} \), and let the contact points have coordinates (in each finger frame) \( c_{f_1}, c_{f_2}, \ldots, c_{f_m} \), respectively. Let the corresponding contact points on the object be given by \( c_{o_1}, c_{o_2}, \ldots, c_{o_m} \) with respect to the object frame \( C_o \). Using the notation of Section II, the matrices \( U_{o_i}, i = 1, \ldots, m \) map the velocity \( (v_i, \omega_i)^T \) of the object frame \( C_o \) to the velocities of the points \( c_{o_i}, i = 1, \ldots, m \), that is
\[
v_i = U_{o_i} \begin{bmatrix} v_o \\ \omega_o \end{bmatrix}
\]
where \( v_i \) denotes the velocity of the \( i \)th contact point. Stacking the \( U_{o_i} \)'s, we get
\[
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_m
\end{bmatrix} =
\begin{bmatrix}
U_{o_1} \\
U_{o_2} \\
\vdots \\
U_{o_m}
\end{bmatrix}
\begin{bmatrix}
v_o \\ \omega_o
\end{bmatrix}
\]
\[
= G \begin{bmatrix}
v_o \\ \omega_o
\end{bmatrix} \tag{3.1}
\]

Note that \( G \in \mathbb{R}^{m \times 6} \) relates the object rotational and translational velocity to the fingertip velocities. A dual relation to (3.1) is obtained by considering the effect of forces \( f_{f_1}, \ldots, f_{f_m} \in \mathbb{R}^3 \) applied at the points \( c_{f_1}, \ldots, c_{f_m} \) respectively, on the object, at the origin of the frame \( C_o \). Using the principle of virtual
work, the desired transformation is found to be

\[
\begin{bmatrix}
  f_0 \\
  \tau_0 
\end{bmatrix} = G
\begin{bmatrix}
  f_1 \\
  f_2 \\
  \vdots \\
  f_m 
\end{bmatrix}.
\]

(3.2)

The origin of the object frame \( C_o \) is frequently chosen to be the center of mass of the object. \( G \) depends both on the location of the contact points and the current object orientation. The matrix \( G \in \mathbb{R}^{\nu \times m} \) is referred to as the grasp matrix: forces in the null space of \( G \) correspond to those forces that can be exerted at the contact points without causing a net force-moment on the object. These are referred to as internal forces. Equations (3.1) and (3.2) provide valid relations if the fingers remain in contact with the object and there is no slipping between the two surfaces. A common way to guarantee no slipping is to ensure that the contact forces lie within the friction cone at each point of contact—that is, the tangential component of the contact force is less than or equal to the coefficient of friction \( \mu \) times the normal component of the contact force. Let \( FC \subseteq \mathbb{R}^1 \) denote the friction cone at the \( i \)th contact point, i.e., if \( f_n^i \) and \( f_\alpha^i \) are the normal and tangential components of \( f^i \), then

\[
FC = \{ f^i \in \mathbb{R}^1 : \| f_\alpha^i \| \leq \mu \| f_n^i \| \}.
\]

(3.3)

Define \( FC = FC_1 \times \cdots \times FC_m \).

For the purpose of grasping, we would like to have the ability to withstand any disturbance force-moment pair on the object. The mathematical characterization of this ability is

\[
G(FC) = \mathbb{R}^\nu,
\]

(3.4)

i.e., the grasp map should map the friction cone onto \( \mathbb{R}^\nu \), so that a given force-moment on the object can be achieved by an appropriate choice of contact forces lying in the friction cone. This property of a grasp has been called grasp stability [6] or force closure [7]. In this paper, we refer to condition (3.4) as the force closure condition.

Internal forces represent the ability to apply tension and compression to an object. In order to be able to firmly grasp an object, it is desirable that the internal forces lie in the interior of the friction cone. Mathematically, this condition may be stated as

\[
\mathfrak{N}(G) \cap \mathfrak{FC} \neq \emptyset
\]

(3.5)

where \( \mathfrak{FC} \) is the interior of the friction cone and \( \mathfrak{N}(\cdot) \) denotes the null space of a matrix. Fig. 2 shows an example of a grasp in which the internal or grasping force (indicated by the arrows) lies outside the friction cone (the dotted lines), so that applying this force to the object will result in sliding or slipping at one or more contact points. When condition (3.5) is satisfied, we can bring any given vector of contact forces into the friction cone by adding a sufficiently large force in the null space of \( G \). In this paper, we refer to condition (3.5) as the prehensibility condition, which will be defined as the ability of a hand to grasp an object, without slipping. One application of this notion is as follows: let \( f_n^i, f_\alpha^i \), \( i = 1, \cdots, n \), be a set of contact forces that result in a certain net force and moment to the object. Consider the case where \( f \in FC \), thereby rendering the possibility of the contacts slipping. Then, if condition (3.5) is satisfied, one can choose a large enough grasping force \( f_r \in \mathfrak{N}(G) \cap FC \) so that the sum of \( f\) and \( f_r \) results in the same net object force and moment and provides a pure rolling contact with no slipping. The relationship between prehensibility and force closure is shown in the following.

Proposition 3.1: Suppose the grasp map \( G \) has full rank. Then prehensibility implies force closure.

Proof: Let \( F_r \in \mathbb{R}^\nu \) be arbitrary and let \( f \in \mathfrak{N}(G) \cap FC \) be the prehensibility condition. Then, since \( G \) has full rank, \( G(F_r + \lambda f) = F_r \) for all \( \lambda \in \mathbb{R} \). Now, since \( f \in FC \) and \( FC \) is a cone, there exists a \( \lambda_0 > 0 \) such that \( G(F_r + \lambda f) \in FC \) for all \( \lambda \geq \lambda_0 \). Since \( F_r \) is arbitrary, \( \mathfrak{N}(FC) = \mathbb{R}^\nu \).

Remarks: Note that \( G \) will have full rank as long as we have at least three noncollinear contact points (at least two distinct contact points in the planar case). Also, the prehensibility condition (3.5) is easier to check than the force closure condition (3.4) (see [7]).

Thus far, we have discussed the kinematics of rolling contact without any mention of the kinematics of the manipulator attached to the rolling fingertip. In a multifingered hand, each finger is, in effect, a manipulator. Consider a multifingered hand with rolling contact at the fingertips. Let the \( i \)th finger have \( n_i \) joints and let \( q_i \in \mathbb{R}^{n_i} \) denote the generalized (linear and rotational) coordinates of the links of the manipulator. The Jacobian matrix \( J(q_i) \in \mathbb{R}^{\nu \times n_i} \) relates the velocity \((\dot{q}_i, \omega_i)^T\) of the \( i \)th finger coordinate frame to the joint velocity \( \dot{q}_i \), as follows:

\[
\begin{bmatrix}
  \dot{q}_i \\
  \omega_i 
\end{bmatrix} = J(q_i) \dot{q}_i.
\]

(3.6)

As in Section II we may express the velocity of the point of contact for finger \( i \) as

\[
\begin{bmatrix}
  v_i \\
  \omega_i 
\end{bmatrix} = U_i J(q_i) \dot{q}_i
\]

(3.7)

The matrix \( J \in \mathbb{R}^{n \times n} \) maps the joint velocities \( \dot{q}_i \) to the velocity at the point of contact. Dual to the relationship (3.7) is the relationship between the fingertip contact force \( f_i \) and the vector of torques \( \tau_i \) at the joints of finger \( i \)

\[
\tau_i = J_i^T f_i.
\]

(3.8)

Aggregating the relationships (3.7), (3.8) for \( i = 1, \cdots, m \) yields

\[
\begin{bmatrix}
  v_1 \\
  \omega_1 \\
  \vdots \\
  v_m \\
  \omega_m
\end{bmatrix} = J \begin{bmatrix}
  \dot{q}_1 \\
  \vdots \\
  \dot{q}_m
\end{bmatrix},
\]

(3.9)

with \( J = \text{diag}(J_1, J_2, \cdots, J_m) \in \mathbb{R}^{\nu \times m} \). When \( J \) is square (each finger has 3 degrees of freedom and invertible (each of the \( m \) manipulators is not in a singular configuration), we can invert (3.9) to get

\[
f_i = (J^{-1})^T \tau_i.
\]

(3.10)

In turn, the force and moment applied to the object are given by

\[
\begin{bmatrix}
  f_0 \\
  \tau_0
\end{bmatrix} = Gf_r + GJ^{-1} \tau.
\]

(3.11)

The nonzero singular values of \( GJ^{-1} \) \( \in \mathbb{R}^{\nu \times m} \) provide a measure of how easy it is to apply forces and moments to the object using the joint torques. Also, since

\[
q = J^{-1} G \begin{bmatrix}
  v_0 \\
  \omega_0
\end{bmatrix}
\]

(3.12)

and the singular values of \( J^{-1} G \) are the same as those of \( G J^{-1} \), the same singular values provide a measure of the joint velocities required to produce a given object velocity. The largest singular
value of $G^\dagger$ may be referred to as a force closure index and the smallest as a manipulability index. These indexes are useful as quasi-static performance measures for comparing grasps (see [6]), for example.

IV. DYNAMICS AND CONTROL

In this section, we consider the problem of controlling the position and orientation of an object in $\mathbb{R}^3$ by the appropriate application of torques at the finger joints. There are, in fact, the following two separate considerations.

1) Tracking—we would like the object center of mass to follow a specified desired trajectory asymptotically.

2) Maintain Contact with No Slipping—over the entire trajectory we need to produce nonzero contact forces which lie within the friction cone, so that contact is maintained and there is no slipping at the contact points.

As above, the position and orientation are denoted by $x_o$ and $R_o$, so that the velocity is $(v_o, \omega_o)^T$ with $v_o = x_o$ and the skew symmetric matrix $[\omega_o \times] = R_o R_o^T$. The dynamics of the object, expressed in the inertial base frame, are given by the Newton–Euler equation as

$$\begin{bmatrix} m_o I & 0 \\ 0 & I_o \end{bmatrix} \begin{bmatrix} \dot{x}_o \\ \dot{\omega}_o \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_o \times I_o \omega_o \end{bmatrix} = \begin{bmatrix} f_o \\ \tau_o \end{bmatrix}$$

where $m_o \in \mathbb{R}^{3 \times 3}$ is the diagonal object mass matrix, $I_o = R_o I R_o^T$ is the object inertia matrix expressed in the base frame ($I_o$ is the constant object inertia matrix expressed in the object frame), and $(f_o, \tau_o)$ are the force and moment applied at the object center of mass.

Specifying an orientation trajectory in terms of the rotation matrix $R_o(t)$ poses two difficulties. First, we must specify twelve quantities such that $R_o$ is orthogonal and differentiable at every time. This problem can be dealt with by specifying the initial orientation and $\omega_o$ at each time. The second, more difficult, problem is determining what the “error” in orientation is at a given time. Since there is no norm that is valid on all of SO(3), we must view this problem locally. Thus, we choose $\gamma_o \in \mathbb{R}^3$ to locally parameterize SO(3) so that $R_o = R(\gamma_o)$. One might for instance, take $\gamma_o$ to be the roll, pitch, and yaw coordinates of the object. Given this parameterization, there exists a linear transformation $P(\gamma_o)$ such that

$$\omega_o = P(\gamma_o) \gamma_o.$$  

Using $\gamma_o$ and $\dot{\gamma}_o$, we can now talk about the error and the error rate of the orientation of the object. An orientation for which $P(\gamma_o)$ is singular will be called an orientation singularity (indeed, this is a singularity of the parameterization).

Let

$$X_o = \begin{bmatrix} x_o \\ \gamma_o \end{bmatrix}$$

denote the position and (local) orientation coordinates of the object. Then, using (4.1) and (4.2), we can express the object dynamics (in the inertial base frame) as

$$M_o \ddot{\rho} + N_o = F_o$$

where

$$\rho_o = \begin{bmatrix} I & 0 \\ 0 & P \end{bmatrix}, \quad F_o = \begin{bmatrix} f_o \\ \tau_o \end{bmatrix}, \quad M_o = \begin{bmatrix} m_o I & 0 \\ 0 & I_o \end{bmatrix}, \quad \text{and}$$

$$N_o = \begin{bmatrix} 0 \\ (P \dot{\gamma}_o) \times I_o P \gamma_o + I_o P \dot{\gamma}_o \end{bmatrix}.$$

Recall from (3.2) that $F_o$ is related to the forces applied by the fingers at the respective contact points by

$$F_o = G f_i,$$

where $f_i = (f^i_1, \ldots, f^i_n) \in \mathbb{R}^m$ is the vector of components of each of the $m$ forces exerted by the fingers at their respective contact points. Combining (4.3) and (4.4), the contact forces needed to generate the object acceleration $\ddot{X}_o$ are given by

$$f_i = G^\dagger (M_o \ddot{\rho} \gamma_o + N_o) + f_i$$

where $G^\dagger = (GG^\dagger)^{-1}$ is the pseudoinverse of $G$ (assuming force closure of the grasp $G$) and $f_i$ is any internal force (belonging to the null space of $G$). The dynamics of each finger have the form

$$M_i(q_i) \ddot{q}_i + N_i(q_i, \dot{q}_i) = \tau_i - J_i^T f_i,$$

where $M_i(q_i) \in \mathbb{R}^{n_i \times n_i}$ is the positive definite moment of inertia matrix for the $i$th finger, $N_i(q_i, \dot{q}_i) \in \mathbb{R}^{n_i}$ is a vector of gravity, Coriolis, and friction terms, and $\tau_i \in \mathbb{R}^{n_i}$ is the vector of input joint torques. (Note that the term $-J_i^T f_i$, in (4.4) is the torque, given by (3.8), due to the force $-f_i$ that the object is applying to the fingertip.) We have assumed, in the above, that the number of joints for each finger is equal to the number of independent forces which can be applied at the contact point (e.g., for three-dimensional manipulation $n_i = 3$). For $m$ fingers, the equations (4.6) may be aggregated to give the dynamics for the hand.

$$M(q) \ddot{q} + N(q, \dot{q}) = \tau - J^T f$$

with

$$M(q) = \text{block diag} (M_1(q_1), \ldots, M_m(q_m)) \in \mathbb{R}^{m \times m},$$

$$N(q, \dot{q}) = (N_1(q_1, \dot{q}_1), \ldots, N_m(q_m, \dot{q}_m))^T \in \mathbb{R}^{m \times 1}$$

and

$$\tau = (\tau^1, \ldots, \tau^m)^T \in \mathbb{R}^{m \times 1}.$$  

Combining (4.5) and (4.7), we get

$$M \ddot{q} + N = \tau - J^T (G^\dagger M_o \ddot{\rho} \gamma_o + N_o + f).$$

The velocity constraint for pure rolling is obtained using (3.1) and (3.7), and making the substitution (4.2) for the chosen parametrization of SO(3), we get

$$J \dot{q} = G^\dagger \ddot{\rho} \gamma_o.$$  

Differentiating (4.9), we obtain the acceleration constraint

$$J \ddot{q} + J \dot{q} = G^\dagger \ddot{\rho} \gamma_o + G^\dagger \dot{\rho} \gamma_o + G^\dagger \ddot{\rho} \dot{\gamma}_o - J \ddot{q}.$$  

Note that the grasp map $G$ is a time varying map since the contact points change as the fingers move on the object. Now, provided that the system does not go through a singular configuration, $J^{-1}$ will exist so that

$$\dot{q} = J^{-1} (G^\dagger \ddot{\rho} \gamma_o + G^\dagger \dot{\rho} \gamma_o + G^\dagger \ddot{\rho} \dot{\gamma}_o - J \ddot{q}).$$  

Using this in (4.8) yields

$$[M J^{-1} G^\dagger + J^T G^\dagger M_o] \ddot{\rho} \gamma_o = \tau - N$$

$$-M J^{-1} (G^\dagger \ddot{\rho} + G^\dagger \dot{\rho} \gamma_o - J \ddot{q}) - J^T (G^\dagger N_o + f).$$

We now propose a control law which allows $X_o(t)$ to track a given desired trajectory $X_o(t)$ asymptotically. It also independently provides us with a nonzero grasping force, so that the contact forces at the fingertips lie within a friction cone and there is no sliding between the finger and object surfaces. The proposed control law for this purpose has the following three terms.
1) A nonlinearity cancellation term of the form
\[ N + MJ^{-1}((G^{T} \tilde{P} + G^{T} \tilde{P})X_{0} - \tilde{J}q) + J^{T}G^{+}N_{g} \]
2) Proportional and derivative error feedback terms to give the system decoupled linear error dynamics
\[ [MJ^{-1}G^{T} + J^{T}G^{+}M_{e}] \hat{P}X_{0} + J^{T}G^{+}K_{x}E + J^{T}f_{s}E \]
where, for example, \( K_{x} = k_{x}I, K_{p} = k_{p}I \) with \( k_{x}, k_{p} \) scalars chosen such that \( s^{2} + k_{x}s + k_{p} \) is Hurwitz. Also \( E = X_{0} - X_{a} \).
3) A term of the form \( J^{T}f_{s}E \), with \( f_{s} \) chosen in the null space of \( G \) so as to keep the contact forces within the friction cones at each point of contact.

Note that terms 1) and 2) are reminiscent of the computed torque control law, in as much as 1) cancels nonlinearities and 2) places the eigenvalues of the closed-loop linear dynamics. The following is the main result of this section.

**Theorem 4.1:** Consider an object being manipulated (with rolling at the contacts) by \( m \geq 2 \) fingers, each having 3 degrees of freedom, with dynamics given by (4.8). Consider the control law
\[ \tau = [MJ^{-1}G^{T} + J^{T}G^{+}M_{e}] \hat{P}(X_{0} + K_{x}E + K_{p}E) + N + MJ^{-1}((G^{T} \tilde{P} + G^{T} \tilde{P})X_{0} - \tilde{J}q) + J^{T}G^{+}N_{g} + J^{T}f_{s}E \]  
(4.13)
with \( f_{s} \) belonging to the null space of \( G \). If the trajectory is such that a) the prehensibility condition is satisfied, b) the fingers avoid configuration singularities, and c) the object does not pass through an orientation singularity, then \( f_{s} \) can be chosen so as to keep the contact forces within the friction cone and the tracking error and error rate converge to zero.

**Proof:** By assumption, both \( J \) and \( \tilde{P} \) are nonsingular over the trajectory so that the control law is well defined. Using (4.13) in (4.12) yields
\[ [MJ^{-1}G^{T} + J^{T}G^{+}M_{e}] \hat{P}(E + K_{x}E + K_{p}E) = J^{T}(f_{s} - f_{s}) \]  
(4.14)
Premultiplying (4.13) by \( G^{T} \) results in
\[ [M_{e} + G^{T}MJ^{-1}G^{T}] \hat{P}(E + K_{x}E + K_{p}E) = 0 \]  
(4.14)
which, for positive definite and \( G^{T}MJ^{-1}G^{T} \) is semidefinite (positive definite, in fact, since \( G^{T} \) is onto), and \( \tilde{P} \) is nonsingular by assumption so that (4.14) implies that
\[ E + K_{x}E + K_{p}E = 0 \].  
(4.15)
Appropriate selection of \( K_{x}, K_{p} \) implies that \( E, E \rightarrow 0 \). Also, since the prehensibility condition is satisfied and all terms remain bounded, we can choose \( f_{s} \) in the null space of \( G \) to keep the contact forces within the friction cone. \( \square \)

V. SIMULATIONS

At first, the planar version of the above law was used to control a planar two-fingered hand, and we present results of one such simulation. Later the control law of (4.13) was tested in a full three-dimensional dynamic simulation of a three-fingered hand. In these simulations, all rolling constraints were strictly enforced so that the simulation could be continued even when the resulting contact forces lie outside the friction cone. The simulation could then be analyzed to see whether the friction cone constraint is violated. Additionally, the range of grasping forces necessary to bring the contact forces into the friction cone could be determined.

A graphics package has been developed to show the motion of the fingers and object through time in a movie. Additionally, the contact force at each contact point is drawn as a line segment showing both magnitude and direction. Viewing these pictures, it is easy to see when the forces stray from the friction cone.

To find a grasping force in \( \Sigma(G) \cap FC \), we have used the following heuristic. Let \( f_{\text{normal}} \) be the force such that each contact point force has unit magnitude in the direction of the object inward normal. Set \( f_{s} \) to the projection of \( f_{\text{normal}} \) onto the null space of \( G \). This projection is accomplished using the projection operator \( (I - G^{T}G) \). If the resulting force has a nonzero component at each contact point which lies within the friction cone, we can bring arbitrary contact forces into the friction cone by adding a sufficiently large multiple of \( f_{s} \) to the contact force. This procedure is a heuristic which has worked well for our examples. The general problem of finding a force in \( \Sigma(G) \cap FC \) as well as determining when the prehensibility condition (3.5) is satisfied is the subject of ongoing research. As representatives of the simulation, we present the results of two simulations, one planar and one three-dimensional.

A. Elliptical Fingerpints on an Elliptical Object
(Three-Dimensional)

A planar two-fingered hand was considered. Each finger consists of two links of unit length with revolute joints and an elliptically-shaped fingertip rigidly attached to the end of the second link. The object to be manipulated is also elliptical in shape. The matrices used in the velocity constraint equation (4.9) are given as follows.

Each elliptical curve has the form
\[ c(\xi) = \begin{bmatrix} a \cos(\xi) \\ b \sin(\xi) \end{bmatrix} \]
(\( \xi \), here, is a parametrization by angle). Thus, \( J = \text{diag}(J_{x}, J_{y}) \) with
\[ J_{x} = U_{ij}J_{y} = \begin{bmatrix} 1 & -b \sin(\xi_{y}) \\ a \cos(\xi_{y}) & -s_{12} - s_{22} - s_{22} - c_{12} + c_{22} \end{bmatrix} \in \mathbb{R}^{2 \times 2} \]
where \( s_{12} = \sin(\theta_{1} + \theta_{2}), c_{22} = \cos(\theta_{2}). \) etc. Also,
\[ G^{T} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}, \]
where
\[ U_{ij} = \begin{bmatrix} 1 & -b \sin(\xi_{ai}) \\ a \cos(\xi_{ai}) \end{bmatrix} \in \mathbb{R}^{1 \times 3} \]
The control law (4.13) was used to control the corresponding dynamic system and a number of frames from the resulting simulation are shown. The desired trajectory of the object center of mass is a circle while the orientation varies as a sinusoid. Fig. 3 shows frames from the simulation with grasping force \( f_{s} \) set to zero. Note that the contact forces often stray from the friction cone. This would normally result in sliding or loss of contact. Fig. 4 contains frames from the simulation run again with the grasping force \( f_{s} \) determined using the heuristic described above. Now, the contact forces remain within the friction cone so that the same result might have been achieved by a real system.

B. Ellipsoidal Fingerpints on an Ellipsoidal Object
(Three-Dimensional)

A hand with three, three-jointed fingers, was considered here. Kinematically, each finger is an elbow manipulator without a wrist, that is, a manipulator with three revolute joints, such that the body axis and shoulder axis are orthogonal, and the elbow axis is parallel to the shoulder axis. An ellipsoidal fingertip is attached at the end of each finger, and the object being manipulated is ellipsoidal in shape. (See Fig. 5.) Ellipsoidal fingertips and an
Fig. 3. Planar rolling with no grasping force.

Fig. 4. Planar rolling with adequate grasping force.

Fig. 5. Three ellipsoidal fingertips rolling on an ellipsoidal object. Each finger has the kinematics of an elbow manipulator without a wrist.

ellipsoidal object were chosen as representative surfaces with non-constant curvature.

Rolling in three dimensions differs from rolling in the two-dimensional case in that it is nonholonomic (see [10]). For example, when the object goes through a continuous periodic motion, the contact points are not guaranteed to be periodic. Using UNIGRAFIX, a three-dimensional graphics package [8] designed at the University of California at Berkeley, we have produced movies of the dynamic simulations of several trajectories. The desired trajectory of the object center of mass shown in Fig. 6 is an ellipse in the vertical plane, with roll-pitch-yaw angles varying as a sinusoid. In addition to showing the position (and the motion in time) of the components of the system, a visual representation of the contact force direction and friction cone is provided for each point of contact. Below each finger is a short stick depicting the contact force direction and a small ring formed by intersecting the boundary of the friction cone with a plane that is parallel to the tangent plane at the point of contact. Thus, if the moving end of the stick lies in the interior of the ring, then the contact force lies within the friction cone. The effectiveness of this display is much more apparent when viewing the motion of the system in real time (as on a SUN™ workstation).

VI. CONCLUSION

In this work, we have considered the manipulation of objects of arbitrary shape by multifingered hands, where the contact between the object and the fingers are rolling contacts. The kinematics, prehensibility, dynamics, and control of these systems have been developed in this paper. Simulations were performed to show the effectiveness of the control scheme.

REFERENCES


Arlene B. A. Cole (S’86) was born in Sierra Leone, West Africa, in December 1959. She received the B.Sc. degree (First Class Honors) in applied mathematics from Warwick University, Warwick, England, in 1979, and the M.Phil degree in control engineering and operation research from Cambridge University, Cambridge, England, in 1981. From October 1980 to December 1981, she worked for Kennedy and Donkin Consulting Engineers, Surrey, England. In January 1982, she took up an appointment as Lecturer at the University of Sierra Leone, Sierra Leone, West Africa, where she remained until August 1984. She is currently completing the Ph.D. degree in electrical engineering and computer science at the University of California, Berkeley. Her research interests are in the area of kinematics, dynamics, and control of robotic manipulators, and also grasp planning for multifingered hands.

Ms. Cole was the recipient of the Sierra Leone Government National Scholarship.

John E. Hauser (S’87) was born in Chicago, IL, on September 26, 1958. He received the B.S.E.E. degree from the U.S. Air Force Academy, Colorado Springs, CO, in 1980 and the M.S. degree in electrical engineering from the University of California, Berkeley, in 1986.

From 1980 to 1984 he flew Air Force jets throughout the United States and Canada participating in active air defense exercises. He is currently completing the Ph.D. degree in electrical engineering at the University of California, Berkeley. His current research interests include nonlinear control of high performance jet aircraft, learning control, robotics, and optimization.

S. Shankar Sastry (S’70-M’80) received the B.Tech. degree from the Indian Institute of Technology, India, in 1977, the M.S. and Ph.D. degrees in electrical engineering in 1979 and 1981, respectively, and the M.A. degree in mathematics in 1980, all from the University of California, Berkeley.

After having been on the faculty of the Massachusetts Institute of Technology, Cambridge, from 1980 to 1982, he is currently a Professor in the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley. His areas of research interests are nonlinear control and systems, adaptive control, kinematics, and control of multifingered robot hands.

Dr. Sastry was an Associate Editor of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL and the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS and is an Associate Editor of the IMA Journal of Control and Information and Large Scale Systems. He received the President of India Gold Medal in 1977, the IBM Faculty Development Award for 1983-1985, and the Presidential Young Investigator Award in 1985.