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EECS 126 — Review Problems

1. A communication channel operates as follows:

At the n-th second, a packet of X_n bits are received at the input of the channel, where X_n is a Poisson random variable of unknown but deterministic rate λ. The random variables X_n are independent.

a) An estimator for λ given $X_1, \ldots, X_n$ is proposed as

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} X_i .$$

Compute the mean square error of this estimator as a function of λ. What happens as $n \to \infty$?

b) Suppose each packet is transmitted with probability $1/2$, independently of all other packets. Let Z denote the number of bits transmitted in the first 3 seconds, and assume $\lambda = 1$. Compute the characteristic function

$$\phi_z(\omega) = E(e^{j\omega z}) .$$

c) Let Z be as above. Compute

$$E(Z|X_1) .$$

2. There are two slot machines. To play, you insert one quarter, and the machine returns either no quarter or two quarters. Of the two machines, Machine A returns two quarters with probability 0.4, and Machine B returns two quarters with probability 0.6.

Assume that the machines are memoryless.

a) Calculate the mean of the number of quarters returned by playing a randomly selected machine once.

b) You randomly pick a machine, and play it for 10 times, and win (i.e., it returns two quarters) 6 times. What is the probability that this is Machine B?

c) Suppose you continue to play the machine of Part (b). Given your experience with it, what is the mean of the number of quarters returned by playing it once more?

d) (Think about it after the exam and before going to Las Vegas!) Suppose you are allowed to play 100 times only, what strategy will give you maximum expected return?

3. A class in probability theory is taking a multiple choice test. For a particular problem on the test, the fraction of the examinees who understand the problem is p; $1-p$ is the fraction that will guess. The probability for answering the problem correctly is 1 for an examinee who understands the problem and $1/m$ for one who guesses.

Compute the probability that an examinee understands the problem given that he has correctly answered it. Clearly define your random experiment, the sample space, events, and associated probabilities.

4. Scott's company (Federal Express) delivers 1 million packages a day. The probability that a package will be damaged in the delivery process is 0.01. Assume that product damages occur independently. What is the approximate probability that more than 10,200 packages will be damaged a day? Clearly define the random variables needed for producing the solutions.

5. Let X and Y be independent, identically distributed RVs with the exponential probability density function

$$f_X(w) = f_Y(w) = \lambda e^{-\lambda w} u(w), \quad w \geq 0$$

where $u(\cdot)$ is the unit step function.

a) Determine the probability density function for $R = \begin{cases} \dfrac{X}{X+Y} & X \geq 0 \text{ and } Y > 0 \\ 0 & \text{otherwise} \end{cases}$

b) Determine the conditional density function of R given $X = x$.

6. A multiplexer combines two independent message streams. The number of message arrivals for each message stream is a Poisson random variable with mean 10 message/second. Compute the probability that more than 600 messages arrive at the multiplexer per minute. Compute it explicitly and use approximation if necessary.

7. Compaq buys modems at the price of $50 each. For each delivered batch of modems, Compaq tests the modems for bit errors by randomly picking 10 modems and transmitting through each modem a test message of 10^3 bits.

If the total bit errors occurred while testing 10 modems exceeds 5, Compaq demands a stiff refund of $25 per modem. If the total bit errors do not exceed 5, for each bit error, Compaq demands a refund of $1 per modem (per bit error).

Suppose a type of modem features a bit error probability (the probability of transmitting one bit incorrectly) of 10^{-4}, and suppose bit error occurs independently. What is the average price this type of modem will command from Compaq? (Use appropriate approximation if necessary.)