1.8 IN SATURATION, \[ I_0 = \frac{1}{2} k' \frac{V_e}{C} (V_{os} - V_e)^2 (1 + \lambda [V_{os} - V_{eff}]) \]

LET \( I_0 = 20 \mu A \), \( V_{os} = V_{eff} \) \( \rightarrow \) \( \frac{1}{2} k' \frac{V_e}{C} (V_{os} - V_e)^2 = 20 \mu A \)

NOW INCREASE \( V_{os} \) BY 0.5V \( \rightarrow \) \( V_{os} = V_{eff} + 0.5V \)

\[ I_0 = 23 \mu A = 20 \mu A [1 + \lambda (0.5V)] \]

\[ \lambda = 0.3 \]

\[ \Gamma_{ds} = \Gamma_0 = \frac{1 + \lambda V_{os}}{\lambda I} \]

\( \frac{I}{\lambda I} = \frac{1}{(0.3) (20 \mu A)} = 166.7 \text{ kS} \)

1.10 FIND \( C_{gs}, C_{gd}, C_{db}, C_{sb} \) FOR A N channel IN SATURATION

USE DEVICE PARAMETERS FROM P6. 59-60 IN J & M

\[ C_{gs} = C_{gs\text{, channel}} + C_{ol} = \frac{2}{3} W L C_{ox} + W C_{ol} \]

\[ C_{gs} = \frac{2}{3} (50 \text{um})(1.2 \text{um})(1.9 \text{FF}) + (50 \text{um})(0.2 \text{FF}) = 86 \text{ FF} \]

WHERE \( C_{gs\text{, channel}} \) IS THE CHARGE IN CHANNEL

CHARGE ASSOCIATED WITH A CHANGE IN \( V_{os} \).

IN SATURATION, THE CHANNEL AT THE DRAIN END IS VERY NARROW, AND EXERTS VERY LITTLE INFLUENCE ON THE CHARGE IN THE CHANNEL OR ON THE GATE. AS A CONSEQUENCE, THE \( C_{gd} \) CAPACITANCE CONSISTS OF A PARASITIC OXIDE CAPACITANCE DUE TO GATE OVERLAP OF THE DRAIN.

\[ C_{gd} = C_{ol} \cdot W = (50 \text{um})(0.2 \text{ FF}) = 10 \text{ FF} \]
$C_{si}$ and $C_{db}$ ARE PARASITIC DEPLETION REGION
CAPACITANCES BETWEEN THE SUBSTRATE AND THE SOURCE & DRAIN
REGIONS. THESE CAPACITANCES ARE PROPORTIONAL TO THE AREA
OF SOURCE AND DRAIN, AS WELL AS THE PERIMETER (SIDEWALL) OF THE
SOURCE AND DRAIN $P+$ (OR $P+$) DIFFUSIONS. $C_{si}$ ALSO INCLUDES
DEPLETION REGION CAPACITANCE FROM THE INDUCED CHANNEL TO THE
BODY, AND ONLY THREE SIDES OF THE DIFFUSION REGION
CONTRIBUTE TO THE SIDEWALL CAPACITANCE (THE CHANNEL SIDE IS
IGNORED). THE DEPLETION REGION CAPACITORS ARE NONLINEAR,
WITH THE GENERAL FORM:

$$C_{j} = \frac{C_{jo}}{\sqrt{1 + \frac{V_{o}}{V_{bi}}}}$$

WHERE $C_{j}$ IS THE CAPACITANCE PER UNIT AREA (OR LENGTH OF
SIDEWALL), $C_{jo}$ IS THE CAPACITANCE AT ZERO BIAS, $V_{o}$ IS THE
REVERSE BIAS ACROSS THE PN JUNCTION, AND $V_{bi}$ IS THE BUILT-IN
POTENTIAL ACROSS THE P-N JUNCTION. SO

$$C_{sb} = C_{j} \left(A_{s} + WL\right) + C_{j-sw} \cdot P_{s} = 2.4 \times 10^{-4} \text{pF} \frac{200 \text{um}^{2} + (50 \text{um})(1.2 \text{um})}{\text{um}^{2}} + 20 \times 10^{-12} \text{pF} (58 \text{um})$$

$$C_{sb} = 62.4 \text{ fF} + 11.6 \text{ fF} = \boxed{74 \text{ fF}}$$

$$C_{db} = C_{j} \cdot A_{d} + C_{j-sw} \cdot P_{d} = 2.4 \times 10^{-4} \text{pF} \frac{200 \text{um}^{2}}{\text{um}^{2}} + 2.0 \times 10^{-12} \text{pF} \text{[58]}$$

$$C_{db} = 59.6 \text{ fF}$$
Assume the output has reached steady state, i.e., $V_{\text{out}} = 1 \, \text{V}$.

**The device is in linear region:**

- $V_{DS} = 0$
- $V_{GS} - V_{T} = 5 - 1 = 4$
- $V_{DS} < V_{GS} - V_{T}$

The channel charge associated with the gate voltage is:

$$Q_{\text{channel}} = -WL \cdot C_{ox} \cdot (V_{GS} - V_{T}) = -10 \mu\text{m} \cdot 0.5 \mu\text{m} \cdot \left(\frac{5 \, \text{pF}}{\mu\text{m}^2}\right) \cdot (5 - 1) = -160 \, \text{fC}$$

The channel charge is composed of electrons, so it is negative.

Once the Xistor turns off, half of the channel charge flows onto the load capacitor. The change in voltage is then:

$$\Delta V = \frac{\Delta Q}{C_L} = \frac{-160 \, \text{fC}}{1 \, \mu\text{F}} = -0.16 \, \text{V}$$
At $t = 0$, $V_{in} = V_{out} = 1\, V$, so the device is in linear region. We can find the channel resistance by taking the partial derivative of the drain current $W/R$ with respect to $V_{ds}$:

$$I_d = k' \frac{W}{L} \left[ (V_{gs} - V_T) V_{ds} - V_{ds}^2 \right]$$

$$(R_{ds})' = \frac{\partial I_d}{\partial V_{ds}} = k' \frac{W}{L} \left[ V_{gs} - V_T - V_{ds} \right]$$

Upon first inspection, the channel resistance might appear to be changing as the output node $V_{out}(t)$ charges up. However, since the step voltage is positive, $V_{out}$ is the source of the transistor, and $R_{ds}$ becomes:

$$R_{ds} = \frac{1}{k'(\frac{W}{L}) [V_T - V_{out} - (V_{in} - V_{out})]} = \frac{1}{k'(\frac{W}{L}) [V_T - V_T - V_{in}]}$$

For $V_{in} = 1.2\, V$:

$$R_{ds} = \frac{1}{\left(\frac{200\, \mu A}{V^2}\right) \left(10\, \text{m}A\right) \left(0.8\, \text{m}^2\right) \left(5 - 1 - 1.2\, V\right)} = 142.9\, \Omega$$

For $V_{in} = 3.1\, V$:

$$R_{ds} = \frac{1}{\left(\frac{200\, \mu A}{V^2}\right) \left(10\, \text{m}A\right) \left(3.1\, V\right) \left(5 - 1 - 3.1\, V\right)} = 444.4\, \Omega$$
Our circuit now simplifies to an RC low pass filter. Evaluate KCL at node \( V_{out} \)

\[
\frac{V_{in}}{R_{ds}} - \frac{V_{out}(t)}{C_L} = \frac{V_{in} - V_{out}(t)}{R_{ds}} = C_L \frac{dV_{out}(t)}{dt}
\]

Separation of variables...

\[
\frac{dt}{R_{ds} C_L \left( V_{in} - V_{out} \right)} = \frac{dV_{out}}{V_{in} - V_{out}}
\]

Assume \( V_{in} \) switches instantaneously at \( t_0 \)

\[
\int_{t_0}^{t} \frac{dV_{out}(t)}{V_{out}(t) \left( V_{in} - V_{out} \right)} = \int_{V_{out}(t_0)}^{V_{out}(t)} \frac{dV_{out}}{V_{in} - V_{out}}
\]

\[
\Rightarrow \quad \frac{t - t_0}{R_{ds} C_L} = \ln \left( \frac{V_{in} - V_{out}(t_0)}{V_{in} - V_{out}(t)} \right)
\]

\[
\Rightarrow \quad V_{out}(t) = V_{in} - \left[ V_{in} - V_{out}(t_0) \right] e^{-\frac{t-t_0}{R_{ds} C_L}}
\]

We want to find

\[
\frac{V_{in} - V_{out}(t)}{V_{in} - V_{out}(t_0)} = 0.01 = e^{-\frac{t-t_0}{R_{ds} C_L}}
\]

Solve for

\[
t - t_0 = -\ln(0.01) R_{ds} C_L
\]

For \( V_{in} = 1.2 \text{ V} \)

\[
t - t_0 = -\ln(0.01)(142.9 \Omega)(1 \text{ pF}) = 658 \text{ ps}
\]

For \( V_{in} = 3.1 \text{ V} \)

\[
t - t_0 = -\ln(0.01)(444.4 \Omega)(1 \text{ pF}) = 2.06 \text{ ns}
\]

Solving for \( V_{out} = 0.99 V_{in} \) is incorrect.
For $g_m$, find average of $\left. \frac{\partial I_p}{\partial V_6} \right|_{V_r \text{ constant}}$.

$$g_{m1,2} = \frac{0.1 - 0.05 A}{-75 V - (-93 V)} = 2.8 \text{ mS}$$

$$g_{m1,2} = \frac{0.1 - 0.2 A}{-75 - (-52)V} = 4.3 \text{ mS}$$

$$g_{m\max} = 3.55 \text{ mS}$$

$$\left( \frac{\partial I_p}{\partial V_p} \right)_{V_6 \text{ const}} = R_{on} = \left( \frac{0.1 - 0.05 A}{3450 - 750 V} \right) = 54 \text{ k\ Ohm}$$

$$A_{V_i} = \left( \frac{\partial V_p}{\partial V_i} \right) = \frac{\Delta V_p}{\Delta V_i} = \frac{3450 - 2500 V}{-75 - (-68) V} = 135$$

The screen is a second coil, with a larger pitch, inserted between the grid and plate, held at a DC bias. The screen reduces the capacitance between the grid and plate (analogous to $C_{gk}$ in CMOS), and improves the frequency response of the vacuum tube.
The impedance of the DC voltage supplies \((V_{DD}, V_{BIAS}, V_{IN})\) to an AC signal is zero. In a small-signal model, this allows us to replace each supply with a short.

First let's calculate the DC bias conditions, assuming both devices are saturated:

\[
I_p = I_n \rightarrow \frac{1}{2} k_p \left( \frac{W}{L} \right) \left( \left| V_{IN} \right| + \left| V_p \right| \right) \left( 1 + \lambda_p \left| V_{OUT} - V_{DD} \right| \right) = \frac{1}{2} k_n \left( \frac{W}{L} \right) \left( V_{GS, n} - V_{TH} \right)^2 \left( 1 + \lambda_n \left| V_{OUT} \right| \right)
\]

From the MOS model:

\[
V_{TH} = \frac{1}{2} V_p, \quad V_p = 0.7 \text{ V}, \quad k'_n = \frac{330 \text{ mA}}{V^2}, \quad k'_p = \frac{100 \text{ mA}}{V^2}, \quad \lambda_n = 0.01 \text{ V}^{-1}, \quad \lambda_p = 0.05 \text{ V}^{-1}
\]

Plug in values, solve for \(V_{OUT} = 2.509 \text{ V}\)

The OP statement from HSPICE gives \(V_{OUT} = 2.5083 \text{ V}\)
CHECK SATURATION ASSUMPTIONS:

\[ V_{DSn} = 2.51 \, V \quad \Rightarrow \quad V_{DSn} > V_{SS} - V_T \]

\[ V_{SS} - V_T = 0.2996 \quad \text{NMOS IS SATURATED} \]

\[ |V_{DS}| = 2.49 \, V \quad \Rightarrow \quad |V_{DS}| > |V_{SS} - V_T| \]

\[ |V_{DS} - V_T| = 0.3 \quad \text{PMOS LOAD IS SATURATED} \]

HSPICE GIVES \( V_{BSATn} = 0.2996 \, V, \) \( V_{BSATp} = 0.3 \, V \)

CALCULATE BIAS CURRENT:

\[ I_{d1n} = \frac{1}{2} k_i n(\lambda_n) (V_{SS} - V_T)^2 (1 + \lambda_n V_{BSn}) \]

\[ I_{d1n} = 15.182 \, \mu A \quad \text{HSPICE GIVES} \quad I_d = 15.1819 \, \mu A \]

LET'S CALCULATE & COMPARE SMALL SIGNAL PARAMETERS TO HSPICE

\[ G_m = G_{mn} = \sqrt{2 I_d n(\lambda_n) (1 + \lambda_n V_{BSn})} = 101.35 \, \mu S \]

HSPICE GIVES \( G_m = 101.3479 \, \mu S \)

\[ R_o = \frac{(1 + \lambda_n V_{BSn})}{\lambda_n I} = 6.752 \, M \Omega \]

HSPICE GIVES \( R_o = \frac{1}{g_{dsn}} = 6.752 \, M \Omega \)
\[ r_{op} = \frac{(1 + \lambda_p V_{sp})}{\lambda_p I} = 1.481 \text{ M}\Omega \]

HSPICE GIVES \[ r_{op} = \frac{1}{g_{ds}} = 1.481 \text{ M}\Omega \]

So the total output impedance is:

\[ R_o = r_{on} \parallel r_{op} = 1.215 \text{ M}\Omega \]

HSPICE GIVES 1.2149 M\Omega

The small signal, low frequency gain is then:

\[ A_v = -g_{m_n} R_o = -123,14 \]

HSPICE GIVES -123,1282

Now we can relate the amplitude of the input \& output sinusoids. The input amplitude was 20 mV.

\[ N_{in} = A_{s} \sin(\omega t) = 0.02 \sin \left[(2\pi \times 10^3) t \right] \]

\[ N_{out} = A_v N_{in} = -2.46 \sin \left[(2\pi \times 10^3) t \right] \quad \text{V} \]

\[ = 2.46 \sin \left[(2\pi \times 10^3) t + \pi \right] \]

\[ V_{out} = V_{out} + N_{out} = 2.51 + 2.46 \sin \left[(2\pi \times 10^3) t + \pi \right] \]
The amplitude of the output sinusoid is 2.46 V. This gives $V_{\text{min}} = 0.05 \text{ V}$. If $V_{\text{max}} = 4.97$, so both the Pmos and Nmos transitions go out of saturation and the waveform clips.
FILE: common_source.sp

* SPICE netlist for "common_source" (generated by MMI_SUE4.1.11)

* start main CELL common_source
* .SUBCKT common_source bias gnd in out vdd
M_0 out in gnd gnd n W='1*1u' L=ln_min ad='arean(1,sdd)' as='arean(1,sdd)' + pd='perin(1,sdd)' ps='perin(1,sdd)'
M_1 out bias vdd vdd p W='3*1u' L=lp_min ad='areap(3,sdd)' + as='areap(3,sdd)' pd='perip(3,sdd)' ps='perip(3,sdd)'
* .ENDS $ common_source

Vin in gnd SIN (0.9996 0.02 10k)
Vdd vdd gnd 5
Vbias bias gnd 4

.include '140model'
.option post=2 nomod

.op
tf v(out) vin
dc vin .98 1.02 0.01
.tran 0.01e-4 5e-4 0

.END

*******************************************************************************
OUTPUT
*******************************************************************************

*****
* file: common_source.sp

*****
* reading file: /share/b/hspice/99.2/99.2/hspice.ini

* spice netlist for 'common_source' (generated by mmi_sue4.1.11)

* start main cell common_source
* .subckt common_source bias gnd in out vdd
m_0 out in gnd gnd n w='1*1u' l=ln_min ad='arean(1,sdd)' as='arean(1,sdd)' + pd='perin(1,sdd)' ps='perin(1,sdd)'
m_1 out bias vdd vdd p w='3*1u' l=lp_min ad='areap(3,sdd)' + as='areap(3,sdd)' pd='perip(3,sdd)' ps='perip(3,sdd)'
* .ends $ common_source

vin in gnd sin (0.9996 0.02 10k)
vdd vdd gnd 5
vbias bias gnd 4

.include '140model'
.model n nmos level=1 vt0 = 0.7 kp = 330u lambda = 0.01
.model p pmos level=1 vt = -0.7 kp = 100u lambda = 0.05

.param lp_min = 1u ln_min = 1u sdd = 5
.param area(n, width, length) = 'width*length*1e-12'
.param area(p, width, length) = 'width*length*1e-12'
.param perip(width, length) = '2u*(width+length)'
.param perin(width, length) = '2u*(width+length)'
.option post=2 nomod

.op
tf v(out) vin
**** operating point information

\[ t_{nom} = 25.000 \quad temp = 25.000 \]

****

**** operating point status is all simulation time is 0.

node = voltage node = voltage node = voltage

+0:bias = 4.0000 0:in = 999.6000m 0:out = -0.5083
+0:vdd = 5.0000

****

voltage sources

subckt
element 0:vin 0:vdd 0:vbias
volts 999.6000m 5.0000 4.0000
current 0. -15.1819u 0.
power 0. 75.9096u 0.

total voltage source power dissipation = 75.9096u watts

**** mosfets

subckt
element 0:m_0 0:m_1
model 0:n 0:p
id 15.1819u -15.1819u
ibs 0.
ibd -25.0828f 24.9172f
vgs 999.6000m -1.0000
vds 2.5083 -2.4917
vbs 0.
vth 700.0000m -700.0000m
vdsat 299.6000m -300.0000m
beta 338.2773u 337.3758u
gam eff 527.6252m 527.6252m
gm 101.3479u 101.2128u
gds 148.1043n 675.0000n
gmb 35.1120u 35.0652u

cdot 2.516e-16 7.566e-16
cgtot 2.369e-16 7.107e-16
cstot 7.395e-16 2.2184f
cbtot 7.652e-16 2.2976f
cgs 2.302e-16 6.906e-16
cgd 1.155e-18 3.442e-18

****

small-signal transfer characteristics

\[ v(\text{out})/\text{vin} = -123.1282 \]

input resistance at \( v(\text{in}) \) = 1.000e20

output resistance at \( v(\text{out}) \) = 1.2149x

\[ A_v = -g_{mn} R_o \]

\[ R_{in} = \infty \]

\[ R_o \]
CLIPPING (NMOS GOING OUT OF SATURATION)