PROBLEM 1

\[ K_n' = V_n C_OX \]
\[ K_p' = V_p C_OX \]

Our small model looks like:

\[ V_{out} \rightarrow 2.5 \text{V} \]

To calculate the operating point assume you bias the output node at \( V_{out} = 2.5 \text{V} \), then solve the current equation for \( V_{s,n} \) and \( V_{s,p} \):

\[ I_n = \frac{K_n'}{2} \left( \frac{V_n}{C_OX} \right) (V_{s,n} - V_T)^2 (1 + \lambda V_{s,n}) \rightarrow V_{s,n} = V_T + \sqrt{\frac{2I_n}{K_n'} \frac{W}{L} (1 + \lambda V_{s,n})} \]

\[ V_{s,n} = 1 + \sqrt{\frac{80\mu A}{200 \mu A (100) [1 + 0.1(2.5)]}} = 1.1788 V_{s,n} \]

Similarly:

\[ V_{s,p} = 1 + \sqrt{\frac{80\mu A}{100 \mu A (200) [1 + 0.1(2.5)]}} = 1.1788 V_{s,p} \]

Note: For quick calculations of \( V_{n,sat,n} \), you don't need to include the \( (1 + \lambda V_n) \) term. I am including it in order to get better agreement with HSPICE.

\[ g_m = \sqrt{2I_n K_n' (V_n)(HAV_{s,n})} = \sqrt{800 \mu A (200 \mu A) (100) [1 + 0.1(2.5)]} = 4.47 \text{mS} \]
\[
\frac{q_{m}}{V_{T}} = \sqrt{\frac{800 \mu A \cdot 100 \mu A}{V_{T}}} \cdot (200 \frac{1+0.1(2.5)}{0.1(400 \mu A)} = 4.47 m \text{m}^2 = q_{m}
\]

\[
\Gamma_{n} = \frac{1 + \lambda V_{os}}{\lambda T} = \frac{1 + 0.1(2.5)}{0.1(400 \mu A)} = 3125 \Omega = \Gamma_{n}
\]

\[
\Gamma_{p} = \frac{1 + \lambda V_{os}}{\lambda T} = 3125 \Omega = \Gamma_{n}
\]

The total impedance looking into the output

Node 1 \( R_{o} = \frac{\Gamma_{p}}{\Gamma_{n}} = 15.625 \text{ K}\Omega = R_{o} \)

Capacitors

NMOS \( C_{gs} = C_{\text{gumax}} + C_{o} = \frac{2}{3} W L \text{Cox} + W \cdot C_{o} \)

\[
= \frac{2}{3} \cdot 5 \frac{\text{FF}}{\mu \text{m}^2} \left(100 \mu \text{m} \cdot 1 \mu \text{m} \right) + 100 \mu \text{m} \left[0.5 \frac{\text{FF}}{\mu \text{m}} \right] = 323.3 \text{ FF} + 50 \text{ FF}
\]

\[
C_{gs} = 383.3 \text{ FF}
\]

\( C_{gd} = C_{o} = 100 \mu \text{m} \left[0.5 \frac{\text{FF}}{\mu \text{m}} \right] = 50 \text{ FF} = C_{gd}, \)
Problem 1 (cont)

\[ C_{db1} = \frac{C_{j0}}{\sqrt{1 + \frac{V_{db}}{V}}} \]

\[ C_{db2} = \frac{C_{j0}}{\sqrt{1 + \frac{V_{db}}{V}}} \frac{1 \text{ff/um}^2}{\sqrt{1 + \frac{\mu}{\rho}} \times [100 \text{um}^2]} = 40.82 \text{ff} \cdot C_{db1} \]

PMOS

\[ C_{db2} = \frac{C_{j0}}{\sqrt{1 + \frac{V_{db}}{V}}} \frac{1 \text{ff/um}^2}{\sqrt{1 + \frac{\mu}{\rho}} \times [200 \text{um}^2]} = 81.6 \text{ff} \cdot C_{db2} \]

\[ C_{gd2} = C_{gd1} \times W = \left[ \frac{0.5 \text{ff}}{\text{um}} \right] \times [200] = 100 \text{ff} \cdot C_{gd2} \]

From the attached space output, we see that our hand calculations match very well.

Note: Why do I specify \( V_{gs, P} \) to four decimal places? We are operating in a high gain region, so for a hypothetical gain of 100, for every

\[ \frac{1}{100} = 0.01 \text{V} \]

by which my input bias voltage \( V_{in} \) is changed, the output changes by 1 V.

\[ A_v = \frac{\Delta V_{out}}{\Delta V_{in}} \rightarrow \Delta V_{out} = A_v \Delta V_{in} = A_v(0.01\text{V}) = 1 \text{V} \]
PROBLEM 2

Let's first define \( R_0 = R_{oc} || r_{oP} \)

\[ C_0 = C_{ab_1} + C_{ab_2} + C_{gd_2} \]

We are interested in the frequency response of:

\[
\begin{align*}
\frac{N_{in}}{V_{in}} & = \frac{1}{1 + \frac{1}{R_0} + \frac{g_mN_{in}}{R_0}} \quad \text{since there is no source impedance} \quad N_{gs} = N_{en} \\
\end{align*}
\]

Summing current at the output node:

\[
\begin{align*}
\frac{sC_{gd}}{R_0} \left( V_0 - V_{in} \right) + g_mN_{in} + \frac{N_{oP}}{R_0} + N_{os} C_0 & = 0 \\
A_v(s) \left( \frac{N_v(s)}{N(s)} \right) & = \frac{\frac{g_m R_0 \left( 1 - \frac{sC_{gd}}{g_m} \right)}{1 + \frac{s}{R_0} \left( C_0 + C_{gd} \right)}}{A(s)} \quad (1)
\end{align*}
\]

Always massage the equation into a form:

\[
A(s) - A_v \left( \frac{1 + \frac{s}{W_{n_1}} \left( 1 + \frac{s}{W_{n_2}} \right)}{1 + \frac{s}{W_{p_1}} \left( 1 + \frac{s}{W_{p_2}} \right)} \right) \quad \text{this allows you to immediately identify the break points in your transfer function}
\]
Problem 2 (Cont.)

We can now rewrite the transfer function, e.g., (1), in terms we are more familiar with.

\[ A_{v0} = -\frac{g_m R_0}{R_o(C_0 + C_{gd})} \quad \text{LOW FREQUENCY GAIN} \]

\[ W_{3dB} = \frac{1}{R_o(C_0 + C_{gd})} \approx \frac{1}{R_o C_0} \quad \text{-3dB FREQUENCY (POLE FREQ.)} \]

\[ W_0 = \frac{g_m}{C_{gd}} \quad \text{ZERO FREQUENCY} \]

\[ W_0 = \frac{g_m}{C_0 C_{gd}} \approx \frac{g_m}{C_0} \quad \text{UNITY GAIN FREQUENCY} \]

Plugging in numbers, \( R_0 = 15.63 \, k\Omega \)

\[ C_0 = C_{dh} + C_{db} + C_{gd} = 22.4 \, \text{fF} \]

\[ A_{v0} = \left[ \frac{4.47 \, \text{mS}}{15.63 \, \text{k}\Omega} \right] \left[ \frac{15.63 \, \text{k}\Omega}{222.4 \, \text{fF}} \right] = \frac{\sqrt{69.87}}{69.87} = A_{v0} \quad \text{SPICE} = 69.8 \]

\[ W_{3dB} = \frac{1}{[15.63 \, \text{k}\Omega][222.4 \, \text{fF}]} = 234.8 \, \text{MHz} \rightarrow 37.4 \, \text{MHz} = W_{3dB} \quad \text{SPICE} = 37.5 \, \text{MHz} \]

\[ W_0 = \frac{4.47 \, \text{mS}}{272 \, \text{fF}} = 16.4 \, \text{GHz} \rightarrow 2.61 \, \text{GHz} = W_0 \quad \text{SPICE} = 2.69 \, \text{GHz} \]

\[ W_2 = \frac{4.47 \, \text{mS}}{50 \, \text{ff}} = 89.4 \, \text{GHz} \rightarrow 14.2 \, \text{GHz} = W_2 \quad \text{SPICE} = 14.2 \, \text{GHz} \]
Problem 2 (cont.)

It's worthwhile to look at the transfer at frequencies between the breakpoints.

We have found that $W_{3db} \ll W_c \ll W_e$.

First look at $\omega \ll W_{3db}$:

$$A(j\omega) = \frac{Av_0 \left[1 - \frac{j\omega}{\omega_c}\right]}{1 + \frac{j\omega}{\omega_{3db}}} \approx Av_0 \quad \text{since} \quad \frac{W}{W_c} \ll 1$$

For $W_c \gg W \gg W_{3db}$:

$$A(j\omega) = \frac{Av_0 \left[1 - \frac{j\omega}{\omega_c}\right]}{1 + \frac{j\omega}{\omega_{3db}}} \approx \frac{Av_0}{j\omega} \quad \text{since} \quad \frac{W}{W_c} \ll 1$$

When does $|A(j\omega)| = 1$? When $W = W_c = Av_0 \omega_{3db} = \frac{g_m}{C_0}$

Now look at $W \gg W_c$:

$$A(j\omega) = \frac{Av_0 \left[1 - \frac{j\omega}{\omega_c}\right]}{1 + \frac{j\omega}{\omega_{3db}}} \approx \frac{W}{\omega_{3db}} + \frac{j\omega}{\omega_c} \quad \text{since} \quad \frac{W}{W_c} \gg 1$$

$$A(j\omega) \approx \frac{g_m}{C_0 + C_0} \cdot \frac{C_0 + C_{gd}}{C_0} \approx \frac{C_{gd}}{C_0}$$
PROBLEM 2 (cont.)

So for high frequencies, $W \gg W_0$, our gain is a capacitive divider that is constant with frequency.

$$\left. A(j\omega) \right|_{W \gg W_0} = A_{\text{H.F.}} = \frac{C_0}{C_0 + 2\pi f \tau} = \frac{50 \text{fF}}{22215 \text{fF}} = \frac{0.184}{A_{\text{H.F.}}}
$$

SPICE $\rightarrow 0.185$

We then expect our frequency response to look like:

\[\text{Diagram showing frequency response with a constant gain of 0.184 at high frequencies.}\]
**Problem 1 and 2**

---

**Problem 2**

---

**Problem 2**

---
PROBLEM 3

Now assume we have Miller multiplication at the input. Our effective input capacitance will rise substantially.

\[ C_{\text{in eff}} = C_{gs} + C_m = 3.93 \, \text{pF} = C_{\text{in eff}} \]

Compared to \( C_0 = 222.4 \, \text{pF} \), we see \( C_{\text{in eff}} \) is almost 18 times larger.

\[ C_m = 50 \, \mu\text{F} \left[ 1 + 69.9 \right] = 3.55 \, \text{pF} \]
PROBLEM 4

**Find the (first) pole of the amplifier.**

We investigated in Problem 3, with \( R_s = 25k \). 

\[ C_{in\,eff} = 3.93 \mu F \]

\[ R_s = 25k \]

\[ C_o = 222.4 \mu F \]

\[ R_o = 15.6 k \]

By comparing the RC delays at the gate drain & gain, it should be clear that your dominant pole is

\[ W_{-3db} = \frac{1}{R_s C_{in\,eff}} \]

\[ N_{gs} = N_{in} \frac{1}{s C_{in}} = \frac{1}{R_s + \frac{1}{s C_{in}}} \]

\[ W_{-3db} = \frac{1}{R_s C_{in\,eff}} = 10.2 \text{ MHz} \rightarrow f_{-3db} = 1.62 \text{ MHz} \]

It turns out that the low-frequency Miller approximation is only valid for calculating the dominant pole \( W_{-3db} \).

For a more accurate analysis (non-dominant poles) we need to go back to the small signal model. [See Tab 15, p. 187]
PROBLEM 5

$C_{\text{in,eff}} = 3.93 \text{ pF}$  

We now want to consider

\[
\begin{align*}
N_i & \quad C_{\text{in}} \quad C_{\text{in,eff}} = C_{\text{in}} + C_{\text{gs}} = 3.93 \text{ pF} \\
\end{align*}
\]

With an ideal voltage drive, there is only a pole at the output node.

\[
A(j\omega) = \frac{A_v}{1 + j\omega \frac{g_m}{g_{m_{in}}}} \quad \Rightarrow \quad \text{3dB} = \frac{1}{R_o (C_v + C_{\text{in,eff}})}
\]

\[
1 + j\omega R_o (C_v + C_{\text{in,eff}})
\]

\[
3\text{dB} = 15.4 \text{ MHz}
\]

\[
f_{-3\text{dB}} = 2.45 \text{ MHz}
\]

This loading scheme should remind you of

\[
\begin{align*}
\text{where } f_{-3\text{dB}} \text{ is the new } -3\text{dB frequency} \\
\text{with the first amplifier loaded with the low frequency effective input capacitance of the second amplifier } [ i.e. C_{\text{in}} - C_{\text{in,eff}} ] \text{. For higher frequencies we need to consider } C_{\text{in,eff}} = C_{\text{gd}} [1 - A(j\omega)] \text{ (see next problem)}
\end{align*}
\]
PROBLEM 6

BASIC AMPLIFIER

\[ A(jw) = \frac{A_{vo}}{1 + \frac{jw}{w_{3db}}} \]

\[ w_{3db} = \frac{1}{R_o C_0} \Rightarrow f_{3db} = 37.4 \text{ MHz} \]

BASIC AMPLIFIER W/ EFFECTIVE CAPACITANCE

Since we are interested in frequency response (as opposed to just \( w_{3db} \) as in Problem 6), we now consider:

\[ A'(jw) = \frac{N_{01}}{N_{in}} \]

Where the importance presented by the second amplifier \( 2z_{in}(jw) \) varies with frequency.

Recall:

\[ A(jw) = \frac{A_{vo}}{1 + \frac{jw}{w_{3db}}} \]

\[ C_{EF} = C_{gd} \left[ 1 - A(jw) \right] \]

Substituting for \( A(jw) \):

\[ C_{EF} = C_{gd} \left[ 1 - \frac{A_{vo}}{1 + \frac{jw}{w_{3db}}} \right] = C_{gd} \left[ 1 - A_{vo} \right] \left[ 1 + \frac{w}{w_{3db}[1 - A_{vo}]} \right] \]
Problem 6

Look at \[ |C_{eff}| \text{ vs. } \omega \]

Now find \[ Z_{in_2} = \frac{1}{j\omega C_m} \]

\[
Z_{in_2} = \frac{1}{j\omega} \cdot \frac{1 + \frac{j\omega}{w_{3db}}}{C_{gd} [1-A_{dc}] \left[ 1 + \frac{j\omega}{w_{3db}(1-A_{dc})} \right]}
\]

- \( Z_{in_2} \) has poles at \( \omega = [0, w_{3db}(1-A_{dc})] \)
- A zero at \( \omega = w_{3db} \)

Let's consider \( Z_{in_2} \) in the range \( w_{3db} < \omega < (1-A_{dc})w_{3db} \)

The term in the numerator simplifies to

\[
1 + \frac{j\omega}{w_{3db}} \approx \frac{j\omega}{w_{3db}} \quad \text{since} \quad \frac{\omega}{w_{3db}} >> 1
\]

Similarly, the pole in the denominator:

\[
1 + \frac{j\omega}{w_{3db}(1-A_{dc})} \approx 1 \quad \text{since} \quad \frac{\omega}{w_{3db}(1-A_{dc})} << 1
\]

\( Z_{in_2} \) then looks like

\[ Z_{in_2} = \frac{j\omega}{w_{3db}} \approx \frac{1}{j\omega C_{gd} [1-A_{dc}]} = \frac{1}{C_{gd} \omega} \]

\[ \text{Looks real} \]

The small signal model we want to solve becomes

\[
\begin{align*}
N_0 & \sim C_{gd} & C_{gs} & N_3 & \downarrow g_{m} & Z_{in_2} & L & C_{gs} & N_2
\end{align*}
\]
Problem 6

Let's use the pole rotating to find $Z_{in2}(j\omega) = \frac{R_0}{\frac{1}{C_{gd}} + Z_{in2}(j\omega)}$

Remember, we evaluate the transfer function far enough away from the break points so that the response is linear. So what about

$$\log|\frac{R_0}{Z_{in1}}| = \log|\frac{1}{R_0} \cdot \frac{1}{Z_{in1}}| = \begin{cases} \log\left|\frac{1}{R_0} + \frac{1}{Z_{in2}}\right| = \log R_0 & \text{for } \omega < \omega_1, \text{ since } |Z_{in1}| \gg R_0 \\ \log|Z_{in1}| & \text{for } \omega > \omega_1, \text{ since } |Z_{in1}| \ll R_0 \end{cases}$$

We apply a similar technique at the breakpoint $\omega_1$, for $Z_{in2} \approx \frac{1}{j\omega C_{gd}}$

A new dominant pole $\omega_1$ is introduced by the low-frequency Miller:

$$\omega_1 = \frac{1}{R_0(C_{gd} + C_{fs})} = \frac{1}{15.625 M\Omega \left[70.9 M\Omega + 227.4 f + 383 f\right]} = 15.4 \text{ M\Omega}$$

Hence gives 2.3 MHz $f_1 = 2.45 \text{ MHz}$ as calculated in Problem 5.
Problem 6

Similarly, \( W_1 \) occurs when \( |Z_{in}| = \left| \frac{1}{jW_1C_0} \right| \)

\[
|Z_{in}(jW)| = \frac{1}{jW} \cdot \frac{jW}{C_{o_d} \cdot (1-A_0) + jW \cdot C_{q_d} \cdot (1-A_0) \cdot s + dB} = \frac{1}{jW C_0} = \frac{1}{W_1 C_0 + C_{q_d}}
\]

\[
W_1 = \frac{C_{q_d} \cdot (1-A_0) \cdot s + dB}{C_0 + C_{q_d}} = \frac{50 \cdot [70.9 \cdot [234.8 \text{ MHz}] = 1.37 \text{ GHz} \rightarrow \frac{f}{W_1} = 218.7 \text{ MHz}}
\]

HSPICE \( \Rightarrow 237 \text{ MHz} \)

The voltage gain \( A_1(jW) = \frac{N_{in}}{N_{out}} = -g_m \cdot Z_{in}(jW) \). See attached plot.

Cascaded Amplifiers w/ Unit Gain Buffer

Since this is an ideal unit gain buffer, the second amplifier cannot load down the first amplifier. Looking at it another way, the input impedance of the buffer is infinite even at all frequencies, so no current \( i_0 \) is pulled from the first stage. The stages become decoupled, and

\[
A(jW) = A_1(jW) \cdot A_2(jW) = \frac{A_{v_o}}{1 + jW / W_{3dB}} \cdot \frac{A_{v_i}}{1 + jW / W_{2dB}} = \frac{A_{v_o}}{\left(1 + jW / W_{2dB}\right)^2}
\]

Notice that \( W_{2dB} \) stays the same; the transfer function rolls off at \( 20 \text{ dB/decade} \) and \( W_1 = A_{v_0} W_{2dB} \) remains unchanged.
PROBLEM 6

CASCaded AMPLIFIERS

\[ A_v = \frac{N_{out}}{N_{in}} = A_1(j\omega) A_2(j\omega) \]

\[ |A_1(j\omega)| \]

\[ |A_2(j\omega)| \]

\[ A_{v^2} \]

CASCADeD
Problem 6

We will learn later that we do not want the non-dominant pole, \( w_p \), less than the unity gain frequency. If it is, it could create 180° of phase shift before the gain of the amplifier drops below 1. In turn, this would result in positive feedback and cause the amplifier to become unstable. Remember, we can change the value of \( w_p \) by increasing \( C_{gs} \) [add extra capacitance]. One convention is to set \( w_p = w_u \), in which the transfer function looks like:

\[
\omega_1 = \frac{C_{gd}(1-A_u)w_u}{C_o + C_{gs}} = \frac{C_{gd}w_u}{C_o + C_{gs}} = w_u
\]

Phase margin = 45°. Remember, \( C_{gs} \) is what we change, let's call it \( C_c \) (for compensation). Then

\[
C_c = C_o + C_{gs}
\]

Is the gain roll off a problem? Well, we never use an amplifier open loop - one always uses negative feedback [remember HW1]. We will learn later that a closed loop amplifier is fairly insensitive to this gain roll-off.
prob6.out

/* 4 - problem 6 */

parameters

.s parameter vin = 100u
.s parameter vpl = 200u
.s parameter lmin = 1u
.s parameter area(width) = 'lu\*width'

.data

vdd vdd vin 0 1.17389 ac=1
vbias vbias 3 3.82111
vbias2 vbias2 3 3.82111
vout vout vin2 1.32111

.endc

.endc

** small-signal transfer characteristics **

\[ V_{\text{out}}/V_{\text{in}} \]

input resistance at \( V_{\text{in}} \) = 1.00e+29

output resistance at \( V_{\text{out}} \) = 15.6242k

** poles (rad/sec) **

-14.4494k 0.0 9.0937k 0.0

-1.4929g 0.0 -237.6495k

** zeros (rad/sec) **

-234.8666g 0.0 -37.3666k 9.0

-09.4450g 0.0 14.3356g 9.0

** notes **

subckt