1. The structure on the left below consists of a rigid body attached to the end of a beam of length \( L \), width \( a \), and thickness \( b \). A force vertical \( F_Y \) acts at a horizontal distance \( r \) from the end of the beam, with \( r \) positive to the right. The tip deflection \( y \) and force are positive in the downward direction.
   a. [2] Write an expression for the deflection and rotation of the tip of the beam as a function of the moment arm \( r \).
   b. [2] Solve for the value of \( r \) that sets the tip deflection to 0.
   c. [2] Solve for the value of \( r \) that sets the tip rotation to 0.
   d. [2] Consider the vertical deflection of the point at which \( F_Y \) is attached. Compare the stiffness of the mechanism in parts b and c to the simple beam (i.e. \( F_Y \) applied at \( r=0 \)).

2. In the structure on the right below, the two beams both have a width \( a \) and thickness \( b \).
   a. [2] Find \( L_2 \) such that the spring constants in the x and y directions are equal.
   b. [2] With that value of \( L_2 \), how much of the x deflection is due to bending in beam 2, and how much is due to rotation of the tip of beam 1?
   c. [4] What is the y deflection due to a force in x, and the x deflection due to a force in y?

3. For a minimum width beam on POLY1 in the polyMUMPS process, assume that \( E=150 \text{ GPa} \) and \( \varepsilon_{\text{max}}=1\% \)
   a. [2] calculate the normal stress and strain due to a 1nN axial force
   b. [2] calculate the axial force, and corresponding deflection, at which the beam will fracture
   c. if the beam is 100um long, with a transverse force of 1nN at the tip,
      i. [2] calculate the bending moment at the base of the beam, \( M(0) \)
      ii. [2] calculate the radius of curvature, \( \rho(x) \), as a function of position along the beam
      iii. [2] calculate the strain and stress as a function of position in the cross-section at \( x=0 \)
      iv. [2] calculate the spring constant and deflection of the tip of the beam, \( y(L) \)
      v. [2] assuming no geometry-based stress concentration, and ignoring any nonlinearity, calculate the transverse force at which the beam will break
      vi. [2] based on our linear model, what is the expected deflection and angular deflection at which the beam will break due to a transverse force?
      vii. [1] Why is our linear model not a good model to use when calculating deflection near the fracture limit?

4. [8] In the SOIMUMPS process,
   a. [4] design a comb drive that will give 10uN of force at 15V, and allow at least +/-10 um of travel.
   b. [2] What is the minimum area of your comb drive, ignoring the anchor?
c. [2] If you wanted to fit it into the smallest possible square, how would that change your design (you may make multiple electrical contacts to fixed electrodes, but there should only be one moving electrode)

5. [4] In the polyMUMPS process, design a spring support that will be very stiff in rotation, fit in a 100x100um² area, and have a stiffness of 10 N/m, while allowing the maximum deflection before fracture.

6. Calculate the deflection of a cantilevered beam of dimension (a, b, L) and density \( \rho \) due to its own weight
   a. If you cut the beam at position \( x \) along its length, what is the weight of the portion from \( x \) to \( L \)? How far is it from \( x \) to the center of mass of that portion? Write an expression for \( M(x) \), the moment on the cross-section at location \( x \).
   b. Write the differential equation with boundary conditions that describe the deflection of the beam, and solve it for \( y(x) \).
   c. For a POLY1 beam in polyMUMPS, what length is necessary to get a deflection equal to the film thickness? What is the aspect ratio (length over thickness) of that beam?

7. For a Tang-style comb drive resonator on POLY1 in polyMUMPS with \( K=1 \)N/m, \( m= 10^{-10} \)kg, \( b=10^{-7} \), and 100 electrostatic gaps per side
   a. Calculate the resonant frequency and the quality factor
   b. Calculate the DC deflection with 1.5, 15, and 150V applied
   c. With a 1.5V sine wave at 1 Hz applied to one comb drive input while the structure is biased at -15V,
      i. Calculate the magnitude of the DC, 1 Hz, and 2 Hz forces
      ii. Calculate the deflection (zero to peak) due to those forces
   d. With the 1.5V sine wave input at the resonant frequency of the structure, and the same DC bias as above
      i. Calculate the deflection at \( \omega_n \) and \( 2\omega_n \).
      ii. If you cut the input frequency in half, calculate the response at \( \frac{1}{2} \omega_n \) and \( \omega_n \)

8. EE247A: For a residual-stress-free clamped-clamped beam of uniform cross section a, b and length L, calculate the deflection at which the nonlinear (cubic) spring force is equal to the linear spring force. How does your answer change if there is residual stress?