Homework Assignment #5  
Due by online submission Friday 10/6/2017 (Saturday 9am)

1. [5] A square silicon solar cell is 10cm on a side and 0.1mm thick. The material has a thermal conductivity of 100 W/m/K. The front of the cell has a heat flow from the sun of 0.1W. The backside of the cell is maintained at 40C. What is the temperature on the front side of the cell?

T =

2. [25] You have made a classic "Tang, Nguyen, Howe" comb drive resonator in the polyMUMPs process using POLY1. You apply a small AC and large DC voltage, and measure a resonant frequency of 10kHz and an amplitude of motion of 1um with a Q of 100 at resonance. With the same amplitude voltages, you sweep the AC signal from 1kHz to 100kHz and record the amplitude of motion at each frequency.
   a. [5] What amplitude do you expect with the AC signal at 1kHz?
      \[ x(1kHz) = \]
   b. [5] What amplitude do you expect with the AC signal at 100kHz?
      \[ x(100kHz) = \]
   c. [5] Some of your structures have folded flexures where you have doubled the length of all of the beams. How do you expect the spring constant and resonant frequency to change for these structures?
      \[ k \quad f_n \]
   d. [10] If the AC voltage is 1.5V zero to peak at 1 Hz, and the DC voltage is 150V, you expect a force at 1Hz with an amplitude \( F_0 \). What other forces do you expect, at what frequency and amplitude?

3. [24] You find that on one of your wafers the foundry made a mistake, and the POLY1 layer is twice as thick as it was on the chip that you measured in the problem above. All other parameters are exactly the same. What impact do you expect this to have on the spring, mass, and damping coefficients \( k, m, \) and \( b \)? Resonant frequency and \( Q \)? Resonant displacement \( x_n \) with the same voltages as used above? Your answers should be of the form "increase by 18" or "decrease by \( \sqrt{7} \)".

<table>
<thead>
<tr>
<th>( k )</th>
<th>( b )</th>
<th>( m )</th>
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<td>( f_n )</td>
<td>( Q )</td>
<td>( x_n )</td>
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4. [15] Four polysilicon resistors are in a Wheatstone bridge with an excitation voltage of 10V. All of the resistors are the same value at room temperature with no strain. Assume that the polysilicon has a temperature coefficient of resistance of 0.1% /C, and a piezoresistive gauge factor of -20.
   a. If one resistor is 1% larger than the rest, what is the magnitude of the bridge output voltage?
      \[ \text{V}_{\text{out}} = \] 
   b. One of the resistors is stretched, increasing its length by 0.1%. What is the percent change in resistance, \( \Delta R/R \)?
      \[ \Delta R/R = \] 
   c. One of the resistors is 100 degrees Celsius warmer than the rest. What is the percent change in its resistance?
      \[ \Delta R/R = \]

5. Beams are 2x2x200um
[35pts, 5 each] You are characterizing a new process and a new material, a thin film. You have made a variety of test structures, and want to find the material properties of the thin film. The film is 1\( \mu \)m thick. You have made several beams that are 10\( \mu \)m wide and 1000\( \mu \)m=1mm long.
   a. You apply 1V across the ends of the beam, and measure a current of 1mA. What is the resistivity of the material?
      \[ \text{Resistivity} = \] 
   b. You flow 1\( \mu \)W of power along the length of the beam, and measure a temperature difference of \( \Delta T \) =10 Kelvin. What is the thermal conductivity of the material?
      \[ \text{Thermal Conductivity} = \] 
   c. When you heat the beam uniformly by 10 Kelvin, the resistance changes by 5 percent. What is the temperature coefficient of resistance?
      \[ \text{TCR} = \] 
   d. When you pull on the beam with a force of 1mN, you measure a change in length of 1\( \mu \)m. What is the modulus of elasticity?
      \[ E = \] 
   e. When you pull on the beam with a force of 1mN, you measure a resistance change of 1 percent. What is the piezoresistive gauge factor?
      \[ G = \] 
   f. You pull on the end of the beam and slowly stretch it. At the point when it is 20\( \mu \)m longer, it breaks. What is the strain limit?
      \[ \varepsilon_{\text{max}} = \]
6. Resistors are nominally 1K, \(V_x=5V\), and \(G=120\) (p-type SCS)

2. [8] You have made a force sensor with a strain gauge at the base of a beam. Three identical resistors make up the rest of a Wheatstone bridge. All resistors are nominally 10k\(\Omega\). The bridge excitation voltage is \(V_e=2V\). The resistors all have a temperature coefficient \(\alpha=10^{-3}/K\), and a gauge factor \(G=-20\).
   
a. If the temperature goes down by 1 C, what is the change in resistance in a single resistor?
   \[ \Delta R= \]

   b. If the strain in the strain gauge increases +1 ppm (\(=10^{-6}\)), what is the change in resistance in that resistor?
   \[ \Delta R= \]

   c. If all resistors heat up by 10 C, what is the change in the bridge output voltage?
   \[ V_{out}= \]

   d. If the product of gauge factor and strain in the strain gauge is \(G_e=0.1\%\), what is the bridge output voltage?
   \[ V_{out}= \]

7. The low frequency deflection is 100nm, and the maximum deflection is 2\(\mu m\) at 5k rad/s.

4. [10] You have made a classic "Tang, Nguyen, Howe" comb drive resonator in a polysilicon process. You apply pure force \(f(t)=F_0\sin(\omega t)\) and sweep the frequency over a wide range. You measure a low frequency deflection of 3\(\mu m\), and the maximum deflection of 90\(\mu m\) occurs at 2,000 rad/s.
   
a. [4] What is the displacement at 20k rad/s? What phase? (write it as \(A\sin(\omega t+\phi)\))
   \[ x_{20k}(t)= \]

   b. [4] What is the displacement at 20 rad/s? What phase?
   \[ x_{20}(t)= \]

   c. [2] You forgot to write down the phase at the maximum deflection. What would you guess it was?
   \[ \phi_{max}= \]

   d. [2] What is the quality factor of the resonator?
   \[ Q= \]

8. \(V_x=5V\) and \(G=-20\) (n-type poly)

7. [6] You have made a cantilevered beam with a single strain gauge at the base. You use that strain gauge in a Wheatstone bridge made with three other resistors, nominally the same value as the strain gauge. Your strain gauge has a gauge factor of 100, and a temperature coefficient of resistance of 2\%/K. The bridge excitation is 1V.
   
c. If your electronics can detect signals down to 1\(\mu V\), what is the minimum detectable strain?
   \[ \varepsilon_{min}= \]