1. In a MEMS differential capacitor with a single moving finger centered between two fixed fingers, the moving finger is biased at $V_f$, and the top and bottom fingers are biased at $\pm V_B$.
   a. Show that the force on the moving finger in the y direction is linear in $V_f$. (i.e. show that $F_y = \beta V_f$, and find $\beta$)
   b. What is the fractional error (deviation from linearity) if the moving finger moves by $g/10$?

2. In an ideal parallel plate actuator of area $A$ and initial gap $g_0$, final gap $g_f < 2/3 g_0$, and linear restoring spring $K$, show that the pull out voltage is $V_{PO} = V_{PI} \frac{3}{2} \sqrt{\frac{1}{3}} \left( \frac{g_f}{g_0} \right) \sqrt{1 - \frac{g_f}{g_0}}$.

3. For the device above, if $V_{PI}=11V$, $V_{PO}=9$, $g_0=3\text{um}$, and the two plates are biased at $V_G$ and $V_B$ respectively.
   a. What is the gap at which the device pulls in?
   b. What is $g_f$?
   c. Sketch the displacement vs. $V_G$ if $V_B=0$. Use arrows to show displacement as the voltage increases above pull-in, and then decreases below pull-out.
   d. Same as part c, except with $V_B=10$ V. Explain how a similar device can be used as a 1 bit memory element. Specifically, label two points on the curve where the device is in a different state for the same value of $V_G$ and explain how one could write a 1 or a 0.

4. In Spencer et al. (reference [1]), an electrostatic gap-closing relay is made with a gate/body overlap area $A_{ov}$, and initial gap $g_0$, a displacement $g_d$ (the amount that it moves before contact), and a spring constant $K$ (see table 1 for specific values of the "current devices" reported in the paper).
   a. Checkout the pictures of the people on the last page – you may recognize some of them
   b. calculate the expected pull-in voltage based on the specs for "current devices"
   c. calculate the expected value for the pull-out voltage based on the same specs
   d. Compare your results from parts a and c to the measured results in Figure 11. Are their results close to what the theory says that should be? What do you think might explain any differences between theory and experiment?
   e. How fast is the mechanical response of these devices today, and how fast do they hope to make them in the “scaled model” future? (see Table 1)

5. In Go & Polhman (reference [2]), they plot measured breakdown voltage vs. electrode gap for many different MEMS structures. The data are taken from many different research papers. As they write, “Fig. 3 highlights the complexity of breakdown at the microscale, where general trends are common but quantitative data are heavily dependant on the electrode geometry, material, and surface roughness.”
   a. In Figure 3, given all of the different data sets, what is the largest safe voltage (no breakdown in any data set) with a 5um gap? With a 1um gap?
   b. In Figure 4c, the data sets from references 19 and 24 are for silicon-silicon gaps, which generally have higher breakdown voltages than metal-metal or silicon-metal gaps. For those two data sets, what is the largest safe voltage with a 5um gap? With a 1um gap?
   c. From Figure 4c, make a table for the data from reference 19, with a column for the gap, the breakdown voltage, and an estimate the electric field at which the devices broke down.

6. In the SOIMUMPS process, you make a gap-closing actuator that is 200um long and 10um wide, with a 2um gap. The actuator is supported by a 1 N/m spring. You apply a DC bias to one side, and a much smaller sinusoidal voltage to the other side.
   a. Calculate the components of the resulting force.
b. Calculate the electrostatic spring constant as a function of DC bias.
c. Sketch the resonant frequency vs. DC bias, with calculated values for the voltage when the
   frequency drops by 10%, and when the frequency goes to zero.
7. [247A] Estimate the pull-in and pull-out time of a gap-closing actuator in air (squeezed-film damping
dominated) and in vacuum (inertia dominated). Either use matlab (or equivalent) to simulate a
   SOIMUMPS gap closer with a 3um initial gap and 1um final gap, or solve the equations in closed form,
or ideally do both. How does the cycle time and the useful mechanical output work depend on your
   choice of spring constant and applied voltage? Does it matter if you run in air or vacuum?
8. [247A] Use Rayleigh-Ritz to estimate the resonant frequency of a cantilever beam. Compare to
   Roark&Young, or your favorite web source of truth.

Applications", IEEE JSSCV46N1, 2011.