\[ U = \frac{1}{2} CV^2 = \frac{1}{2} \epsilon \frac{A}{x} V^2 = \frac{1}{2} \epsilon \frac{tx_0}{g_0 - z} V^2 \]

\[ F_z = -\frac{dU}{dz} = \frac{1}{2} \epsilon \frac{tx_0}{(g_0 - z)^2} V^2 \quad \text{from one side of the comb} \]

Total \[ F_z = \frac{1}{2} \epsilon tx_0 V^2 \left[ \frac{1}{(g_0 - z)^2} - \frac{1}{(g_0 + z)^2} \right] \]

7.2

To find spring constant, take coefficient of 1st term in Taylor expansion.

(Find the linear term, that is.)

\[ F_z^0(z) = \frac{1}{2} \epsilon tx_0 V^2 \left[ \frac{-2(-1)}{(g_0 - z)^3} - \frac{-2}{(g_0 + z)^3} \right] \]

\[ F_z^0(0) = K_e = \frac{1}{2} \epsilon tx_0 V^2 \frac{4}{g_0^3} = \frac{2 \epsilon tx_0 V^2}{g_0^3} \]

Positive spring constant means it's unstable.

Now, we will combine the non-linear electrostatic spring with a linear spring that has twice the stiffness of the electrostatic spring about \( z = 0 \), and a spring that has half the stiffness.

Twice \[ F_z = \frac{1}{2} \epsilon tx_0 V^2 \left[ \frac{1}{(g_0 - z)^2} - \frac{1}{(g_0 + z)^2} \right] - 2 \left( \frac{2 \epsilon tx_0 V^2}{g_0^3} \right) z \]

Half \[ F_z = \frac{1}{2} \epsilon tx_0 V^2 \left[ \frac{1}{(g_0 - z)^2} - \frac{1}{(g_0 + z)^2} \right] - \frac{1}{2} \left( \frac{2 \epsilon tx_0 V^2}{g_0^3} \right) z \]
In class, we derived that pull-in occurs when \( g = \frac{2}{3} g_0 \).

So, when \( V = V_{pi} \), \( g = \frac{2}{3} g_0 \), displacement of \( \frac{1}{5} g_0 \).

Balance mechanical and electrostatic forces:

\[
k \gamma = \frac{1}{2} \varepsilon t W V^2 \frac{1}{g^2}
\]

\[
\frac{E a^3}{L^3} \cdot \frac{1}{3} g_0 = \frac{1}{2} \varepsilon t W V_{pi}^2 \frac{1}{(\frac{2}{3} g_0)^2}
\]

\[
V_{pi}^2 = \frac{8 E g_0^3 a^3}{27 W L^3 \varepsilon}
\]

Pick data points where \( g_0, L, W \) are constant, then

\[
V_{pi}^2 = \frac{8 E g_0^3}{27 \varepsilon W L^3} \left( a + da \right)^2
\]

\[
\frac{\text{da}}{(\text{Constant})}
\]

Plot \( V_{pi}^2 \) vs. \( a^3 \). Fit points to a linear curve.

Slope is \( \frac{8 E g_0^3}{27 \varepsilon W L^3} \), so you can extract value of \( E \).

X-intercept gives \( da^3 \).

Similarly, find \( g_0 \) by holding \( a, L, W \) constant.
First find mechanical spring constant

\[ \text{clamped-clamped} = \text{clamped-free} + \text{clamped-free} \]

\[ \begin{align*}
\text{We know for a clamped-free beam, } & F = \frac{d^3}{3EI} \Rightarrow k = \frac{3EI}{d^3} \\
\text{Spring formula } & \Rightarrow k = \frac{3EI}{d^3} \\
\text{clamped-free beam of length } & \frac{L}{2} \Rightarrow k = \frac{3EI}{(\frac{L}{2})^3} = \frac{24EI}{L^3} \\
\text{clamped-clamped beam of length } & L \\
\text{equivalent to two } & \text{ beams in series } \Rightarrow k = \frac{12EI}{L^3} \\
\text{2 beams in series } & \Rightarrow k_{eq} = \frac{1}{2} k \\
\text{parallel } & \Rightarrow k_{eq2} = 2 k_{eq} = k \\
\end{align*} \]

- Mechanical Spring Constant 
  \[ k = \frac{12EI}{L^3} = \frac{Ea^3t}{L^3} \]
  \[ I = \frac{a^4}{12} \]