Homework 2

Due Friday (5pm), Feb. 8, 2013

Please turn the homework in to the drop box located next to 125 Cory Hall (labeled EE 42/100). Make sure to clearly label your Name, Student ID, Class, and Discussion sections on the homework.

1) The most efficient way to do this problem is with node voltage technique since there is just one node to analyze there is just one equation and one unknown. If we use KCL at node A and KVL in the two loops we will have 3 equations for 3 unknown currents in the branches (and two variables per equation) which is more laborous to solve though very possible. Using the node voltage technique we want to write all the currents in terms of $V_A$, the only unknown extraordinary node voltage. We have:

$$
\text{by KVL: } -V_A + I_1 2k + 8 = 0 \quad \rightarrow \quad I_1 = \frac{V_A - 8}{2k}
$$

$$
-V_A + 4 + I_2 (4k + 8k) = 0 \quad \rightarrow \quad I_2 = \frac{V_A - 4}{12k}
$$

$$
\rightarrow \quad I_x = \frac{V_A}{2k}
$$

In time you won’t need KVL to get these, you know them from ohms law.

KCL at node A: $I_1 + I_2 + I_x = 0$

$$
\frac{V_A - 8}{2k} + \frac{V_A - 4}{12k} + \frac{V_A}{2k} = 0
$$

Multiplying by 12k we get:

$$
6V_A - 48 + V_A - 4 + 6V_A = 0
$$

$$
V_A = \frac{52}{13} = 4V \quad I_x = \frac{V_A}{2k} = 2mA
$$

$$
\text{by KVL: } -V_{AB} + 4 + I_2 4k = 0
$$

$$
V_{AB} = 4 + I_2 4k = 4 + \frac{V_A - 4}{12k} 4k = 4 + 0 \cdot 4k
$$

$$
V_{AB} = 4
$$

You can go through the entire analysis again or it’s ok to recognize that if we had done the problem using KCL/KVL it would be obvious that none of the currents would change since these equations were independent of the ground.
location which means the voltage difference between any two nodes would not change. What we call zero volts has just increased by 8V so all node voltages would decrease by 8V in order to maintain the relative voltages between each node (including ground). Thus neither \( V_{AB} \) nor \( I_x \) would change but \( V_A \) decreases by 8V. Let’s do nodal the analysis anyway remembering that \( V_A \) is now referenced to the new ground:

\[
\text{by KVL: } -V_A + 4 + I_2(4k + 8k) - 8 = 0 \rightarrow I_2 = \frac{V_A + 4}{12k}
\]

\[
-8V + I_x(1k + 1K) - 8 = 0 \rightarrow I_x = \frac{V_A + 8}{2k}
\]

(Compared to part a) there is 8V added everywhere \( V_A \) appears

\[
\text{KCL at node A: } I_1 + I_2 + I_x = 0  \\
\frac{V_A}{2k} + \frac{V_A + 4}{12k} + \frac{V_A + 8}{2k} = 0
\]

Multiply by 12k: \( 6V_A + V_A + 4 + 6V_A + 48 = 0 \rightarrow V_A = -52/13 = -4V \]

\[
I_x = \frac{V_A + 8}{2k} = 2mA, \quad V_{AB} = 4 + I_22k = 4 + \frac{V_A + 4}{12k}2k = 4 + 0 \rightarrow V_{AB} = 4V
\]

Again, you could state that \( V_A \) would decrease from that in part a) by the amount of \( I_x1k \) but we’ll pretend we don’t know that. This is a bit tricky with node analysis but doable. If you wanted, you could add a node at C and have 2 equations to solve but it isn’t necessary since we know the current from ground to C is still \( I_x \). Ground doesn’t sync current since current doesn’t care where we decided to label 0V.

\[
\rightarrow I_x = \frac{V_A}{1k}
\]

\[
\text{KVL: } -V_A + I_12k + 8 - I_x1k = 0, \text{ substitute } I_x, \text{ gives: } -V_A + I_12k + 8 - V_A = 0 \rightarrow I_1 = \frac{2V_A - 8}{2k}
\]

\[
-V_A + 4 + I_2(4k + 8k) - I_x1k = 0, \text{ substitute } I_x, \text{ gives: } -V_A + 4 + I_212k - V_A = 0 \rightarrow I_2 = \frac{2V_A - 4}{12k}
\]

\[
\text{KCL at A: } I_1 + I_2 + I_x = 0 \rightarrow \frac{2V_A - 8}{2k} + \frac{2V_A - 4}{12k} + \frac{V_A}{1k} = 0 \rightarrow 12V_A - 48 + 2V_A - 4 + 12V_A = 0
\]

\[
\rightarrow V_A = \frac{52}{26} = 2V, \quad I_x = \frac{V_A}{1k} = 2mA
\]

\[
\text{KVL: } -V_{AB} + 4 + I_22k = 0 \rightarrow V_{AB} = 4 + I_22k = 4 + \frac{2V_A - 4}{12k}2k = 4 + 0 \cdot 2k \rightarrow V_{AB} = 4V
\]
Using voltage divider, \( V_{out} \) can be written as a function of \( \Delta R \):

\[
V_{out} = \left( \frac{R_x}{R_x + R_o} \right) \cdot 10 = \left( \frac{R_o + \Delta R}{R_o + \Delta R + R_o} \right) \cdot 10 = \left( \frac{R_o + \Delta R}{2R_o + \Delta R} \right) \cdot 10
\]

We can also solve for \( R_x \):

\[
\frac{V_{out}}{10} = \frac{R_x}{R_x + R_o}
\]

\[
\frac{V_{out}}{10} \cdot R_x + \frac{V_{out}}{10} \cdot R_o = R_x
\]

\[
\frac{V_{out}}{10} \cdot R_o = R_x \cdot (1 - \frac{V_{out}}{10})
\]

\[
R_x = \frac{\frac{V_{out} \cdot R_o}{10 - V_{out}}}{\frac{V_{out}}{10}} = \frac{V_{out} \cdot R_o}{(10 - V_{out})}
\]

\[
R_o + \Delta R = R_o \left( 1 + \frac{\Delta R}{R_o} \right) = R_o (1 + \epsilon) = \frac{V_{out} \cdot R_o}{(10 - V_{out})} \rightarrow \epsilon = \frac{V_{out}}{(10 - V_{out})} - 1
\]

When \( \epsilon = 0 \) and \( R_o = 10k \) we get \( V_{out} = 5V \) (Typical voltage divider). Plugging in \( V_{out} = 5.001V \), we get \( \epsilon = 4 \cdot 10^{-4} = 4 \text{ parts per 10,000 or 400 parts per million} \).

\( \Delta R = R_o \cdot \epsilon = 4 \Omega \) so \( R_x \) can be measured with accuracy of \( \pm 2 \Omega \).
Alternate solution:

\[ V_{out} = \left( \frac{R_o + \Delta R}{2R_o + \Delta R} \right) \cdot 10 = \frac{(1 + \frac{\Delta R}{R_o})}{(2 + \frac{\Delta R}{R_o})} \cdot 10 = \frac{(1 + \varepsilon)}{(1 + \frac{\varepsilon}{2})} \cdot 5 \], where \( \varepsilon \equiv \frac{\Delta R}{R_o} \)

We can use Taylor’s series approximation to approximate the denominator. Recall that Taylor’s series allows us to approximate the equation of the form: \( \frac{1}{(1+x)} \approx 1 - x \).

Thus, we have

\[ \frac{1}{(1+\frac{\varepsilon}{2})} \approx 1 - \frac{\varepsilon}{2} \]

\[ \Rightarrow V_{out} = (1 + \varepsilon) \cdot \left( 1 - \frac{\varepsilon}{2} \right) \cdot 5 = (1 - \frac{\varepsilon}{2} + \varepsilon + \frac{\varepsilon^2}{2}) \cdot 5 \]

Since \( \frac{\varepsilon^2}{2} \) is a very small number, we can neglect it.

\[ \Rightarrow V_{out} \approx (1 - \frac{\varepsilon}{2} + \varepsilon) \cdot 5 = (1 + \frac{\varepsilon}{2}) \cdot 5 \]

When \( \varepsilon = 0 \), \( V_{out} = 5V \). Adding the smallest resolution of the voltmeter (0.001V) to this and recalculating the \( \varepsilon \):

\[ 5.001 = (1 + \frac{\varepsilon}{2}) \cdot 5 \Rightarrow \varepsilon = 4E-4 \Rightarrow \Delta R = \varepsilon \cdot R_o = (4E-4)(10k) = 4 \Omega \]

3) A Wheatstone-Bridge sensor is pictured below:

Using voltage division,

\[ V_1 = \left( \frac{R_o}{R_o + R_x} \right) \cdot V_{Supply} = \frac{V_{Supply}}{2} \]

and
\[ V_2 = \left(\frac{R_x}{R_x + R_0}\right) \cdot V_{\text{Supply}} = \left(\frac{R_0 + \Delta R}{R_0 + \Delta R + R_0}\right) \cdot V_{\text{Supply}} = \left(\frac{R_0 + \Delta R}{2R_0 + \Delta R}\right) \cdot V_{\text{Supply}} \]

Then,

\[ V_{\text{out}} = (V_2 - V_1) = \left(\frac{R_0 + \Delta R}{2R_0 + \Delta R}\right) \cdot V_{\text{Supply}} = \frac{V_{\text{Supply}}}{2} \]

\[ = \frac{(2R_0 + 2\Delta R - 2R_0 - \Delta R)\cdot V_{\text{Supply}}}{(2R_0 + \Delta R)\cdot 2} \]

\[ = \frac{(\Delta R)\cdot V_{\text{Supply}}}{(4R_0 + 2\Delta R)} \]

\[ = \frac{(\Delta R)\cdot V_{\text{Supply}}}{4R_0\left(1 + \frac{\Delta R}{2R_0}\right)} \]

If \( \frac{\Delta R}{R_0} \ll 1 \), we simply the equation to:

\[ V_{\text{out}} = \frac{(\Delta R)\cdot V_{\text{Supply}}}{4R_0} = \frac{V_{\text{Supply}}}{4} \cdot \frac{\Delta R}{R_0} \]

With \( V_{\text{Supply}} = 10\text{V} \),

\[ V_{\text{out}} = 2.5 \cdot \left(\frac{\Delta R}{R_0}\right) \]

Accounting for the gain, \( A=10,000 \) of the amplifier we get,

\[ V_{\text{Amp}} = V_{\text{out}}' = A \left(2.5 \cdot \left(\frac{\Delta R}{R_0}\right)\right) = 25,000 \cdot \left(\frac{\Delta R}{R_0}\right) \]

Rearranging the terms, we have:

\[ \left(\frac{\Delta R}{R_0}\right) = \frac{V_{\text{out}}'}{25,000} \]

If \( V_{\text{out}} \) has a smallest resolution of 1mV,

\[ \left(\frac{\Delta R}{R_0}\right) = \frac{10^{-3}}{25,000} = 4 \times 10^{-8} \]

Thus, we can measure the accuracy up to [4 parts per 100 million or 40 parts per billion]

We can find \( \Delta R = 4 \times 10^{-8}R_0 = 0.4\text{ m}\Omega \) so \([R_x \text{ can be measured with accuracy of } \pm 0.2\text{ m}\Omega]\)

The corresponding circuit schematic of a Wheatstone bridge with the amplifier is shown below:
Similarly, we can solve for \( R_x \) in terms of \( V_{out} \):

\[
V_{out} = V_2 - V_1 = \left( \frac{R_x}{R_x + R_0} \right) \cdot V_{Supply} = \frac{V_{Supply}}{2}
\]

\[
V_{out} = \frac{(2R_x - R_0) V_{Supply}}{(R_x + R_0) \cdot 2}
\]

\[
V_{out} = \frac{(R_x - R_0) V_{Supply}}{(R_x + R_0) \cdot 2}
\]

\[
V_{out} (R_x + R_0) \cdot 2 = (R_x - R_0) \cdot V_{Supply}
\]

\[
2V_{out}R_x + 2V_{out}R_0 = R_x V_{Supply} - R_0 V_{Supply}
\]

\[
2V_{out}R_0 + R_0 V_{Supply} = R_x (V_{Supply} - 2V_{out})
\]

\[
R_x = \frac{R_0 (2V_{out} + V_{Supply})}{V_{Supply} - 2V_{out}} = \frac{R_0 (\frac{V_{Amp}}{Gain} + V_{Supply})}{V_{Supply} - 2 \frac{V_{Amp}}{Gain}}
\]

4)

a) Finding the equivalent resistance across two nodes is the same as attaching a voltage source across a node as shown below and finding \( \frac{V}{I} = R_{eq} \) where \( I \) is the current that the voltage source must supply and not the current in the branch from a to b. In fact, we can always find \( R_{eq} \) this way though it will usually be far more time consuming. In any case, this was just to show that no current flows through either 1kΩ resistor and thus they can be removed from the circuit.
Now we can work backwards starting by combining the 3 series resistors from nodes c to e to f to d in series (the same current flows through all 3) and continue from there.

\( \text{b) The easiest way to find } R_{eq}(g,h) \text{ is to recognize that it just so happens that } R_{eq}(e,f) = R_{eq}(a,b) \text{ which we already found in part a) to be } 4k \, \Omega. \text{ It isn’t always the case that } R_{eq} \text{ looking from the left side of a circuit to the right is the same as } R_{eq} \text{ from right to left, but based on these resistor values it just so happens that the resistor configuration is the same. The network is then,} \)

\( \text{c) If we apply a test voltage across nodes c&d, the current that would have to be supplied by the voltage source (not the current flowing in the wire branch from c to d) would be} \)
split into three parts. Again, the 1k resistors can be removed. We can work left to right and and right to left closing in on nodes c and d to get.

\[ R_{eq}(c,d) = \frac{12k}{12k/12k} = 4k \Omega \]

d) Though it can be done, it is a bit tricky to work left to right or right to left since both sides contain either node g or b. Another way, is to redraw the circuit from g to b with all the resistors horizontal as below with the 12k resistor not shorted out yet. To do this, we would follow the path of current as it goes from g to b.

You could have started at b and worked to g it wouldn’t matter. This is a tough circuit to find \( R_{eq} \) and is often used in the context of Y-delta transformations (though it can be solved with nodal analysis). Luckily, we said the 12k \( \Omega \) resistor between nodes c and d is shorted.

After redrawing in a more intuitive way, \( R_{eq} \) is found by combining the series and parallel combinations appropriately.
The equation for an ideal diode assumes that $V_d$ is the voltage from the anode (base of triangle) to the cathode (side with line through it). This also defines the direction of current through the diode $I_d$. The circuit in each case is re-drawn below with the appropriate terminals and current directions through the diode labeled. This is similar to HW1 problem 5) in which the positive and negative terminals have been defined.

5) The equation for an ideal diode assumes that $V_d$ is the voltage from the anode (base of triangle) to the cathode (side with line through it). This also defines the direction of current through the diode $I_d$. The circuit in each case is re-drawn below with the appropriate terminals and current directions through the diode labeled. This is similar to HW1 problem 5) in which the positive and negative terminals have been defined.

\begin{align*}
\text{a)} & \quad \text{KVL gives, } V_d = 9V. \text{ Substituting into the diode equation gives,} \\
& \quad I_d = 10^{-32} (e^{-9V/26mV} - 1) = -I_s = -10^{-32}A. \\
& \quad I_d = -10^{-32}A. \text{The diode is off because it is reverse biased.} \\
\text{b)} & \quad \text{Now } V_d = +9V, \text{ so } I_d = 10^{-32} (e^{9V/26mV} - 1) \rightarrow I_d = 2.15 \cdot 10^{118}A. 
\end{align*}
The LED is hot because there is ideally an enormous amount of power in a small volume. In reality the 9V battery will never source 9V at 2.15 \times 10^{11} \, \text{A} due to the source resistance of the battery and the current will be much, much less.

If we were to account for the source resistance of the battery, the circuit would look like the circuit below where \( V_{batt} = V_d \) is the voltage we would actually measure across the battery and \( R_S \) is the source resistance. In reality, \( V_{batt} \) will drop to be much closer to the diode forward voltage of \( V_F \sim 1.8V \). For example, if \( R_S \) is just 10\( \Omega \), we would get \( I_d = 700 \, \text{mA} \) and \( V_{batt} = 1.9V \)

![Circuit Diagram](image)

c) Since the resistor and diode are in series, they have the same current. KVL of the part c) circuit gives,

\[
KVL: V_d = 9 - I_d \cdot 10M
\]

\[
I_d = \frac{9 - V_d}{10M}
\]

Method 1 (Calculator friendly like what might be used on exam!):

The most calculator friendly way which avoids explicitly calculating exponentials or logarithms starts from the previous equations:

\[
I_d = I_s \left( e^{V_d/V_T} - 1 \right) \quad \text{and} \quad I_d = \frac{9 - V_d}{10M}
\]

The 1st equation can be re-written by assuming \( e^{V_d/V_T} \gg 1 \) and using an exponential version of the change of base formula:

\[
e^{x} = 10^{x/\ln(10)} = 10^{x/2.3}
\]

When substituted for \( I_d \) gives the more user friendly equations:

\[
I_d = 10^{-32} \cdot 10^{V_d/60mV} \quad \text{and} \quad I_d = \frac{9 - V_d}{10M}
\]

Equation1 and Equation2

The first equation shows the 60mV per decade rule, that is, every time \( V_d \) increases by 60mV, \( I_d \) increases by a factor of ten. The algorithm for finding \( I_d \) is:

#1) Guess an \( I_d \) (start with \( I_d \) (max) = 9V/10M~1\mu A)

#2) Calculate value of \( V_d \) that would produce \( I_d \) using the Equation1 60mV/decade rule. (i.e if we guess \( I_d = 10^{-6} \, \text{A} \) then \( 10^{V_d/60mV} \) must be \( 10^{32-6} = 10^{26} \) so \( V_d = 26 \cdot 60\, \text{mV} = 1.56V \)

#3) Using the new value of \( V_d \) calculate \( I_d \) using Equation2.

#4) Go back to #2 using the order of magnitude \( I_d \) from #3.
#5) Continue until the result of #3 no longer changes from one iteration to the next meaning we have found \( V_d \) with sufficient accuracy. The final \( I_d \) is the final result from #3.

The result in this case is shown below:

<table>
<thead>
<tr>
<th>( I_d ) (Guess)</th>
<th>( V_d ) (Using 60mV/Decade)</th>
<th>( I_d ) (Equation2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0E-06</td>
<td>(32-6)*60mV=1.56V</td>
<td>7.440E-07</td>
</tr>
<tr>
<td>1.0E-08</td>
<td>(32-8)*60mV=1.44V</td>
<td>7.560E-07</td>
</tr>
<tr>
<td>1.0E-08</td>
<td>Done</td>
<td>Done</td>
</tr>
</tbody>
</table>

This result gives \[ I_d = 760nW \] and \( V_d = 1.44V \). Note, if we take this value of \( V_d \) and substitute it into the equation \( I_d = I_s (e^{V_d/kT} - 1) \) our current will be significantly off due to the exponential dependence of \( I_d \) on \( V_d \).

Using a solver we would get, \[ V_d = 1.548898V \text{ and } I_d = 745nA \] which we are close to.

Typically for an LED to be bright, it should be emitting about 1mW or so. The LED would be on but it would be barely lit since it is at best (100% efficiency) emitting \( \sim 1\mu W \) of power.

**Method 2:**

We can invert the equation: \( I_d = I_s (e^{V_d/kT} - 1) \) to get \( V_d \) in terms of \( I_d \).

\[
V_d = V_T \ln \left( \frac{I_d}{I_s} + 1 \right)
\]

Substituting into the KVL equation and using \( V_T=26mV \) gives:

\[
\Rightarrow I_d = \frac{9-26mV \ln \left( \frac{I_d}{I_s} + 1 \right)}{10M} \quad \text{or LHS = RHS}
\]

We could use the change of base formula: \( \ln(x) = \log(x)/\log(e) = \log(x)/0.43 \) and assume \( I_d/I_s \gg 1 \) to rewrite the diode voltage as:

\[
V_d \approx 60mV \cdot \log \left( \frac{I_d}{I_s} \right)
\]

Which shows that \( V_d \) increases by 60mV for every factor of ten increase in \( I_d \)(60mV per decade rule). Substituting the change of base into the equation for \( I_d \) gives.

\[
\Rightarrow I_d = \frac{9-60mV \log \left( \frac{I_d}{I_s} \right)}{10M} \quad \text{or LHS = RHS}
\]

This is a transcendental equation with no analytical solution for \( I_d \). The hand right side (RHS) of the equation will decrease as \( I_d \) increases. We can guess an \( I_d \) and if RHS>LHS we should increase the value of \( I_d \) for the next guess and vice versa if RHS<LHS.

The algorithm is:

#1) Guess an \( I_d \) (Can start with \( I_d(max)=11V/10M\Omega \sim 1\mu A) \)

#2) Calculate \( \Delta = RHS - Guess \)

#3) Make \( \text{NewGuess} = \text{OldGuess} + \Delta \) and go back to #2)
#4) Repeat process until *Guess* is within one significant figure of *RHS*.

Overall, it took only 3 guesses to arrive at $I_d$ as shown below:

<table>
<thead>
<tr>
<th>LHS(Guess)</th>
<th>RHS</th>
<th>Delta=RHS-LHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0E-06</td>
<td>7.4E-07</td>
<td>-2.6E-07</td>
</tr>
<tr>
<td>7.4E-07</td>
<td>7.5E-07</td>
<td>7.7E-10</td>
</tr>
<tr>
<td>7.5E-07</td>
<td>7.5E-07</td>
<td>Done</td>
</tr>
</tbody>
</table>

So we get $I_d = 750 \text{nA}$ and $V_d = 9 - I_d \cdot 10M = 1.5V$. Again, we cannot expect to substitute $V_d = 1.5V$ into the equation $I_d = I_s \left( e^{V_d/V_T} - 1 \right)$ and get an accurate answer for $I_d$. Our current is very close to the actual current though.

**Method 3 (most painful, only use when require accurate $V_d$):**

This is the most brute force method that also gives the most accurate value of $V_d$.

Since the problem didn’t ask for $V_d$ to great precision this isn’t necessary but it is the first way I tried that is unfortunately most time consuming. We can solve for $V_d$ rather than $I_d$. Substituting the equation $I_d = I_s \left( e^{V_d/V_T} - 1 \right)$ into the KVL equation gives,

$$ V_d = 9 - 10^{-32} \left( e^{V_d/26mv} - 1 \right) \cdot 10M \text{ or } LHS=RHS $$

We can use the exponential version of the change of base formula: $e^x = 10^{x/\ln(10)} = 10^{x/2.3}$ and assume $e^{V_d/26mv} \gg 1$ to rewrite this as:

$$ V_d = 9 - 10^{-32} \cdot 10^{V_d/60mv} \cdot 10M \text{ or } LHS=RHS $$

Again, this is a transcendental equation. We cannot follow the same algorithm to get LHS=RHS as we did in Method2 because the RHS is an exponential function of $V_d$ therefore it doesn’t converge. Instead a more ad-hoc guessing scheme was used which requires more significant digits on $V_d$ in order to get LHS=RHS to within 1 significant digit. As $V_d$ increases, the RHS decreases so if RHS>LHS the guess for $V_d$ was increased.
So we get $V_d = 1.5488V$ and $I_d = 10^{-32}(e^{1.5487V/26mv} - 1) \Rightarrow [I_d = 740nA]$

Note, If you just went with $V_d=1.6V$. You would get $I_d=5.32\mu A$ which is off by a factor of 7. This is a case where significant figures matter for $V_d$ which makes it advantageous to solve for $I_d$ directly using Method1 or Method2.

**d)** We have:

$$I_d = 10mA = 10^{-32}(e^{V_d/26mv} - 1) \approx 10^{-32} \cdot 10^{V_d/60mv}$$

The factor $10^{V_d/60mv}$ must equal $10^{32-2} = 10^{30}$ so,

$$V_d = 30 \times 60mV = 1.8V$$

KVL around the part d) loop gives,

$$V_d = 9 - I_d \cdot R$$

$$\Rightarrow R = \frac{(9 - V_d)}{I_d} = \frac{(9 - 1.8)}{10mA} = 720 \Omega$$
In order to determine which voltage source is absorbing or delivering power, power needs to be calculated for each of the sources. If power turns out to be negative, then it is supplying power. If power turns out to be positive, then it is absorbing power. Since \( P = IV \), we need to determine the current \( I \). To this end, we can start writing down KVL equation.

KVL in a counter clock-wise direction gives:

\[-10 - V_2 + 5 = 0\]
\[-10 - (I \times 2) + 5 = 0\]

\[I = -\frac{5}{2} \text{ A} \]

(NOTE 1: What does this negative current mean?? Based on the circuit schematic which was used to construct our KVL equation, the current direction is drawn such that it is flowing from the 5V towards the 10V. The negative sign in our answer indicates that the real current direction is actually flowing from the 10V towards the 5V instead.)

\[P_{5V} = -IV = -(-\frac{5}{2}) \times 5 = 12.5 \text{ W} \rightarrow 5V \text{ is absorbing power}\]

\[P_{10V} = IV = (-\frac{5}{2}) \times 10 = -25 \text{ W} \rightarrow 10V \text{ is supplying power}\]

(NOTE 2: A curious student might ask how come when calculating the power for the 5V supply you are using \( P = -IV \) instead of just \( P = IV \). Another student, who vaguely remembers reading something about Passive sign convention, explains to the curious student that the negative sign in the equation is there to account for the fact that current is drawn to be entering the negative terminal (or, equivalently, leaving the positive terminal) of the 5V supply in the circuit schematic. The curious student then thanks another student for the help, and promises that he/she won’t make this same mistake on the test.)
Writing KVL in a clock-wise direction gives:

\[-5 + V_2 - 10 = 0\]
\[-5 + (I \times 2) - 10 = 0\]

\[I = \frac{15}{2} = 7.5 \text{ A}\]

\[P_{5V} = - IV = - (7.5)(5) = -37.5 \text{ W} \rightarrow 5V \text{ is supplying power}\]

\[P_{10V} = - IV = - (7.5)(10) = -75 \text{ W} \rightarrow 10V \text{ is supplying power}\]

Writing KVL in a clock-wise direction gives:

\[5 + V_2 - 10 = 0\]
\[5+ (I \times 2) - 10 = 0\]

\[I = \frac{5}{2} = 2.5 \text{ A}\]

\[P_{5V} = IV = (2.5)(5) = 12.5 \rightarrow 5V \text{ is absorbing power}\]

\[P_{10V} = - IV = - (2.5)(10) = -25 \text{ W} \rightarrow 10V \text{ is supplying power}\]
7. 

**Solution:** (a) At nodes \( V_1 \) and \( V_2 \),

\[
\begin{align*}
\text{Node 1:} & \quad \frac{V_1 - 16}{1} + \frac{V_2}{1} + \frac{V_1 - V_2}{1} = 0 \\
\text{Node 2:} & \quad \frac{V_2 - V_1}{1} + \frac{V_2}{1} + \frac{V_2}{1} = 0
\end{align*}
\]

(1)

(2)

Simplifying Eqs. (1) and (2) gives:

\[
\begin{align*}
3V_1 - V_2 &= 16 \\
-V_1 + 3V_2 &= 0
\end{align*}
\]

(3)

(4)

Simultaneous solution of Eqs. (3) and (4) leads to:

\[
V_1 = 6 \text{ V}, \quad V_2 = 2 \text{ V}.
\]

(b)

\[
V_R = V_1 - V_2 = 6 - 2 = 4 \text{ V}
\]

\[
I = \frac{V_2}{1} = \frac{2}{1} = 2 \text{ A}.
\]

8. 

**Solution:** At nodes \( V_1 \), \( V_2 \), and \( V_3 \),

\[
\begin{align*}
\text{Node 1:} & \quad \frac{V_1}{2} + \frac{V_1 - V_2}{3} - 4 = 0 \\
\text{Node 2:} & \quad \frac{V_2 - V_1}{3} + \frac{V_2 - 48}{2} + \frac{V_2 - V_3}{6} = 0 \\
\text{Node 3:} & \quad \frac{V_3 - V_2}{6} + \frac{V_3}{4} + 4 = 0
\end{align*}
\]

(1)

(2)

(3)

Simplification of the three equations leads to:

\[
\begin{align*}
5V_1 - 2V_2 &= 24 \\
-2V_1 + 6V_2 - V_3 &= 144 \\
-2V_2 + 5V_3 &= -48
\end{align*}
\]

(4)

(5)

(6)

Simultaneous solution of Eqs. (4)–(6) leads to:

\[
V_1 = \frac{84}{5} \text{ V}, \quad V_2 = 30 \text{ V}, \quad V_3 = \frac{12}{5} \text{ V}.
\]

Hence,

\[
I_e = \frac{V_2 - V_3}{6} = \frac{30 - 12/5}{6} = 4.6 \text{ A}.
\]
To find the Thevenin equivalent circuit, we need to solve for the $V_{TH}$ and $R_{TH}$.

**Finding $V_{TH}$:**

Using node analysis method, we can label 2 extraordinary nodes: ‘Node 1’ and ‘Node 2’.

At Node 1 (Let’s assume that current flowing out of the node is considered to be positive):

$$-3 + \frac{V_1}{1} + \frac{V_1 - V_2}{2} = 0$$  \hspace{1cm} (1)

At Node 2, we recognize that $V_2$ is simply equal to 5V. Thus, no need to perform KCL at node 2.

$$V_2 = 5V$$  \hspace{1cm} (2)

Substituting $V_2$ into Eq (1), we can solve for $V_1$:

$$-3 + \frac{V_1}{1} + \frac{V_1 - 5}{2} = 0$$

$$V_1 = \frac{11}{3} \text{ V}$$

To find $V_{ab}$, we need to find the voltage at node a and b since $V_{ab} = V_a - V_b$.

What is $V_a$? It turns out that $V_a$ is actually the same node that we initially labeled as node 1.

Thus, $V_a = V_1 = \frac{11}{3} \text{ V}$.

$V_b$ can be found by using voltage divider between the 4Ω and 2Ω resistors which is connected to the 5V supply.

$$V_b = \left(\frac{2}{2+4}\right) \cdot 5V = \frac{5}{3} \text{ V}$$

Therefore, $V_{ab} = V_a - V_b = \frac{11}{3} - \frac{5}{3} = 2 \text{ V}$

**Finding $R_{TH}$:**

To find $R_{TH}$ of the circuit, we can first deactivate the independent sources in circuit and then solve for $R_{eq}$ as seen through the port (a,b).

To **deactivate the current source**, we can replace it with an open-circuit. To **deactivate the voltage source**, we can **short it out** by replacing with a wire. The circuit should appear as followed:
We see that 2Ω and 1Ω are in parallel, as well as the 4Ω and 2Ω.

\[ R_{TH} = R_{eq} = \frac{1}{2} + \frac{4}{2} = \frac{1 \cdot 2}{1 + 2} + \frac{4 \cdot 2}{4 + 2} = 2 \, \Omega \]

Therefore, Thevenin equivalent circuit can be represented as:

\[ R_{TH} = 2 \, \Omega \]
\[ V_{TH} = 2 \, V \]

To find Norton equivalent circuit, we can use source transformation:
Thus, we have:

\[ I_N = \frac{V_{TH}}{R_{TH}} = 1 \text{ A} \]

(b) With the applied voltage source \( V \), the circuit becomes:

Current \( I \) can be expressed in terms of voltage \( V \) as:

\[ I = \frac{V - V_{TH}}{R_{TH}} = \frac{V - 2}{2} \text{ [A]} \]
Solution:

\[ \frac{V}{4} + \frac{V}{6} + \frac{V - 2}{3} = 4 \]

Hence, \( V = \frac{56}{9} \) V.

\[ V_{Th} = V_{oc} = V - 2 = \frac{56}{9} - 2 = 4.22 \text{ V.} \]

Suppressing the sources:

\[ R_{Th} = \frac{4}{3} + 2.5 = 3.83 \Omega \]

Thévenin equivalent circuit: