HW8 Solutions, ee42/100, 13sp

1) bits to represent
60: ceil(log2(60)) = 6 bits that’s the expression to use with google, but by hand it’s easy to remember that $2^{10} = 1024 \approx 1000$. So one million is 20 bits, one billion is 30 bits and so on. As long as you can handle the single-digit powers of two, the rest is easy.
3600: $2^{10} = 1024$. $2^2 \times 2^{10} = 4096 \rightarrow 12$ bits
3600*24=86,400: $2^7 \times 2^{10} = 128 \times 1024 \rightarrow 17$ bits
1 month = $2.6\times10^9$; $2^2 \times 2^{10} \times 2^{10} = 4$ million \rightarrow 22 bits
1 year = $\pi \times 10^7$ (Feynman’s approximation): $2^5 \times 2^{10} \times 2^{10} = 32$ million \rightarrow 25 bits
1 millennium = 1000 years, so need 10 more bits \rightarrow 35 bits
14 billion years = $44 \times 10^{16}$s = $440 \times (10^3)^5 \rightarrow$ 440 is covered by 9 bits, each factor of 1000 is 10 bits, so 9+10*5 = 59 bits

Grader: 1 pt each. No credit if the answer is not an integer.

2) a) $V_{IH} = 0.65*V_{CC}$; $V_{IL} = 0.35*V_{CC}$.
   b) $V_{OH} = V_{CC} - 0.3$; $V_{OL} = 0.3V$.

Grader: 1pt each, 4 pts total.

3) samples will look like $\sin(2\pi*1.1kHz \times nT)$
where n is an integer, and T=1/(sampling frequency) = 1ms
a)

<table>
<thead>
<tr>
<th>N</th>
<th>$\sin(2\pi*1.1k \times n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

b) no, this doesn’t look like a 1.1kHz sine wave. It has been aliased to a lower frequency.
c) the samples look like half of a period of a sine wave. Half of the period is 5ms, so the period is 10ms, so the frequency looks like 100Hz.

We’d get the same result if we write
$\sin(2\pi*1.1kHz \times 0ms) = \sin(2\pi*(1+0.1)kHz \times n ms) = \sin(2\pi*(1+0.1)k \times n ms)$

which looks like a sine wave at 10% of the sampling frequency.

Grader: 3 pts for the table in part a (they just need the 6 numbers in the right hand column)
1 pt. for part b. They just need to say “no”.
2 pts for part c.
4.)

A. The Nyquist sampling theorem states that if $f_{\text{sample}} > 2 \times \text{max}\text{(frequency component in signal)}$ then the signal can be reconstructed. Here $f_{\text{sample}} = 3\text{Hz}$ so the largest signal frequency that can be reconstructed is: $f_{\text{Nyquist}} = 1.5\text{Hz}$

Grader: They can just write $1.5\text{Hz}$. 2pts

B. Since $60\text{Hz}$ is a multiple of the sampling frequency, the worst case is for the sample to be taken at the peak of the input sinusoid at all times. If the sampling begins at $t=0$, the worst case, is a noise signal of $10mV \sin(2\pi 60t + 90^\circ)$. This gives a digital output of DC voltage at $\pm 10\text{mV}$. Grader: they can just write "DC voltage at $10\text{mV}$". 2pts

C. The anti-aliasing filter should be a low pass filter in order to prevent sampling of high frequency signals above the Nyquist frequency. This is achieved with a capacitor from the analog output terminal to ground as shown below.

To find $R_{\text{th}}$ as seen by the load, all independent sources are zeroed out and the equivalent resistance as seen by the load is found. The circuit is:

With the assumption that the variable resistor is $\sim 2K\Omega$ this gives:

$R_{\text{th}} = (2K/2K) = 1K\Omega, \quad \omega_p = 2\pi (1\text{Hz}) = 1/R_{th}\tau \longrightarrow C = 1/(2\pi R_{th}) = 160\mu F$

Grader: 1 pt for getting the Thevenin resistor right. 1 pt for the equation relating $C$ and $\tau$, and 1 pt for getting the right answer.

D. To find the transfer function, the resistive divider circuit is replaced by it’s Thevenin equivalent as seen by the next stage. $R_{\text{th}}$ was found above. We know that $V_{\text{th}} = V_{\text{oc}}$. However, $V_{\text{oc}}$ was given in the problem as the output voltage of the analog circuit with no
filter capacitor i.e. no load. Thus \( V_L = 10mV + 10mV \sin(2\pi t) + 10mV \sin(2\pi 60t) \) and the thevenin equivalent of the analog circuit is:

![Thevenin Equivalent Circuit](image)

Turning to the phasor domain: \( H(jw) = \frac{V_{filter}}{V_{th}} = \frac{Z_C}{Z_C + R} = \frac{1}{1+jw/w_p} \)

\[
|H(jw)| = \frac{1}{\sqrt{(1+(w/w_p)^2)}} \quad \Rightarrow \quad \angle H(jw) = -\tan^{-1}(w/w_p).
\]

The circuit can be analyzed by superposition so that the ith component of the filter output is: \( V_{filter(i)} = V_{th(i)} |H(jw_i)| \sin(w_i t + \varphi_i + \angle H(jw_i)) \)

We get:

\[
V_{filter} = 10mV + \frac{10mV}{\sqrt{2}} \sin(2\pi t - 45^\circ) + \frac{10mV}{60} \sin(2\pi 60t - 90^\circ)
\]

where it was assumed that the sampling starts at \( t=0 \) and now we assume that the noise has a worst case phase shift of 0.

Grader: 1pt for each term. The 60Hz term should have a phase shift of -90 from what they started with.

E. Since the ADC samples at 3Hz, the sampling period is \( T_s = 1/3 \) s and samples are taken at times of \( t = nT_s = \frac{n}{3} \) where \( n \) is 0,1,2,integers. The digital output of the ADC is then:

\[
V_{Dig} = 10mV + \frac{10mV}{\sqrt{2}} \sin\left(2\pi \frac{n}{3} - 45^\circ\right) + \frac{10mV}{60} \cos\left(2\pi 60 \frac{n}{3}\right).
\]

The last term aliases down to DC. Re-substituting \( n=3t \) into the second term gives:

\[
V_{Dig} = 10mV + \frac{10mV}{\sqrt{2}} \sin(2\pi t - 45^\circ) + 10mV /60
\]

The 60Hz noise has been knocked down a lot (by a factor of 60), which may or may not be enough to make the user happy. The 1Hz signal that the user is commanding has been phase shifted by 45 degrees, or 0.125 seconds. That's probably enough to make someone mad.

Grader: 1 pt for each term in the Voltage. They can be in terms of time or \( n \). 1 pt for some discussion of how the user interprets that signal. (4 pts total for part E)

5.) \( \text{#atoms} = [\text{density}] \times [\text{volume}] = \left[ \frac{8 \text{atoms}}{(0.543 \cdot 10^{-9})^3} \right] \times [1 \times 10^{-6}]^3 \)

\[ \text{#atoms} = 50 \text{ billion} \]

2 pts. 1 for setup, 1 for getting the right answer.
6.) The diamond-shaped arrangement of atoms resembles that of Silicon as viewed in the \(<100>\) direction or it can also be only the closest visible atoms in the \(<110>\) direction highlighted in red below. As the diamond pattern looks a bit elongated in the vertical direction, it is most likely the \(<110>\) direction, which fits with the text.

There are about 28 atoms at the base.

Note that the \(<100>\) direction is the vector perpendicular to the \((100)\) plane and the same goes for the \(<110>\) direction.

The apparent bottom three atoms for each view are highlighted below.
In the $<100>$ and $<110>$ direction the Pythagorean Theorem yields an apparent spacing of $0.543/\sqrt{2}$ nm.

The width of the fin is 28 multiplied by the apparent atomic spacing which gives:

\[10.8 \text{ nm}\]

Grader: 1pt for getting a number of atoms close to 28
1pt for coming up with a spacing that involves the numbers 0.543nm and either 1/2 or 1/sqrt(2), giving answers in the neighborhood of 10nm.