Name and SID:

There are four short questions and six slightly longer problems. Answer on these sheets. Show your work. Good luck.

**Question 1 (6%).** Give an example of a probability space \( \{ \Omega, \mathcal{F}, P \} \) and three events \( A, B, C \) that are pairwise independent but not mutually independent.

**Question 2 (6%).** Let \( \{ \Omega, \mathcal{F}, P \} \) be the probability space that corresponds to rolling a balanced die. Give an example of an event \( A \) in that probability space that is independent of itself.
Question 3 (8%). Let $X$ be exponentially distributed with mean 1. Calculate
\[ E(\max\{X^2, X + 1\}) \]
Question 4 (6%). Let $(X, Y)$ be the coordinates of a point picked randomly and uniformly in the set $S = \{x, y|0 \leq x, y \leq 1 \text{ and } y \geq x^2\}$. Calculate $E(XY)$. 
Problem 1 (8%). \( X \) and \( Y \) are two independent random variables that are exponentially distributed with mean 1 and 2, respectively. Calculate \( P(X > Y) \).
Problem 2 (6%). Let $X$ and $Y$ be two points picked independently and uniformly in the square $[-1, 1]^2$. Let $Z$ be the square of the distance between $X$ and $Y$. Calculate $E(Z)$. 

Problem 3 (10%). Let $X, Y$ be independent and uniformly distributed in $[0, 1]$. Find $\text{cov}(X, Y)$. 
**Problem 4 (8%).** Let $X, Y$ be independent and uniformly distributed in $[0, 1]$. Let $V = 2X + 3Y$ and $W = X + 2Y$. Find the joint density $f_{V,W}(v, w)$ of $(V, W)$. 
Problem 5 (12%). Let $X = B(n, p)$. That is, $P(X = m) = \frac{n!}{m!(n-m)!}p^m(1-p)^{n-m}$ for $m = 0, 1, \ldots, n$. Calculate $\text{var}(X)$.
Problem 6 (10%). Assume that a good light bulb has an exponentially distributed lifetime with mean 1000 hours. Assume also that a defective light bulb has an exponentially distributed lifetime with mean 500 hours. You buy a light bulb. The odds that the bulb is defective are 5%. Assume that the light bulb burns out after $h$ hours. Find the maximum value of $h$ such that the probability that the bulb is defective given that information is at least 50%.