EE126: Problem Set # 1

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Assigned: January 21, 2003 — Due: January 28, 2003

Problem 1. a. Express in the form $a + bi$: $\sin(-1 + i)$.
   b. Solve the equation $e^z = \frac{-1-i}{\sqrt{2}}$.
   c. Find the real part, imaginary part, argument, and absolute value of all solutions of the equation $z^2 + z + 1 = 0$.

Problem 2. a. Prove Pascal's Identity $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.
   b. Prove the Binomial Theorem $(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$ by induction on $n$.

Problem 3. a. How many ways can $n$ books be placed on $k$ distinguishable shelves if:
   i. the books are indistinguishable?
   ii. the books are distinguishable and the position on the shelves matter?
   b. A shelf holds 12 books in a row, how many ways are there to choose 5 books such that no two adjacent books are chosen?

Problem 4. Solve $\int_0^1 x^n e^x dx$.

Problem 5.

\[ A = \{1 \leq n \leq 200|3 \text{ divides } n\} \]
\[ B = \{1 \leq n \leq 200|5 \text{ divides } n\} \]
\[ C = \{1 \leq n \leq 200|7 \text{ divides } n\} \]

Find $|A \cup B \cup C|$

Problem 6. Let $A$ be a nonempty set of real numbers which is bounded below. Let $-A$ denote the set of all $-x$ where $x \in A$. Prove that $\inf A = -\sup(-A)$.