Problem 1. Choose a number randomly between 1 and 999999 inclusive, all choices being equally likely. What is the probability that the digits sum up to 23? For example, the number 7646 is between 1 and 999999 and its digits sum up to 23 \((7+6+4+6=23)\).

Problem 2. a. Let \(A\) and \(B\) be independent events. Show that \(A^c\) and \(B\) are independent.
   b. Let \(A\) and \(B\) be two events. If the occurrence of event \(B\) makes \(A\) more likely, then does the occurrence of the event \(A\) make \(B\) more likely? Justify your answer.
   c. If event \(A\) is independent of itself, show that \(P(A)\) is 1 or 0.
   d. If \(P(A)\) is 1 or 0, show that \(A\) is independent of all events \(B\).

Problem 3. Let \(A_1, A_2, \ldots, A_n, n \geq 2\) be events. Prove that \(P(\bigcup_{i=1}^{n} A_i) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n)\).

Problem 4. A man has 5 coins in his pocket. Two are double-headed, one is double-tailed, and two are normal. The coins cannot be distinguished unless one looks at them.
   a. The man shuts his eyes, chooses a coin at random, and tosses it. What is the probability that the lower face of the coin is heads?
   b. He opens his eyes and sees that the upper face of the coin is a head. What is the probability that the lower face is a head?
   c. He shuts his eyes again, picks up the same coin, and tosses it again. What is the probability that the lower face is a head?
   d. He opens his eyes and sees that the upper face is a head. What is the probability that the lower face is a head?

Problem 5. Let \(\Omega = \{1, 2, 3, 4\}\) and let \(\mathcal{F} = 2^\Omega\) be the collection of all the subsets of \(\Omega\). Give an example of a collection \(\mathcal{A}\) of subsets of \(\Omega\) and probability measures \(P_1\) and \(P_2\) such that
   i. \(P_1(A) = P_2(A), \forall A \in \mathcal{A}\).
   ii. The sigma-field generated by \(\mathcal{A}\) is \(\mathcal{F}\). (This means that \(\mathcal{F}\) is the smallest sigma-field of \(\Omega\) that contains \(\mathcal{A}\).)
   iii. \(P_1\) and \(P_2\) are not the same.