Name and SID:

There are seven problems. Answer on these sheets. Show your work. Good luck.

**Problem 1 (10%).** Give an example of a pair of random variables \((X, Y)\) that are uncorrelated and not independent.

**Problem 2 (10%).** Give an example of a pair of random variables \((X, Y)\) that are not independent and are such that \(E[X|Y] = E(X)\).
Problem 3 (10%). Is it possible for a pair of random variables \((X, Y)\) to be such that \(E[X|Y] > X\) for all \(Y\)? Explain your answer.

Problem 4 (10%). Let \(X, Y, Z\) be independent and uniformly distributed on \([-1, 1]\). Calculate \(E[X + Y|X + Y + Z]\).
Problem 5 (15%). Let $X, Y, Z$ be independent and equally likely to take the values $\{-2, -1, 0, 1, 2\}$. Calculate $L[X + 2Y \mid X + Y, Y + Z]$. 
Problem 6 (25%). Let $X, Z$ be independent with $P(X = 0) = 0.4, P(X = 1) = 0.6,$ and $Z = N(0, 1)$. Find the MLE and the MAP of $X$ given $Y = X + (1 + X)Z$. 
Problem 7 (30%). For $x = 0, 1$, given $X = x$, $Y$ is exponentially distributed with mean $\mu(x)$, for $x = 0, 1$ where $0 < \mu(0) < \mu(1)$.

a. Find $\hat{X} = g(Y)$ that maximizes $P[\hat{X} = 1|X = 1]$ subject to $P[\hat{X} = 1|X = 0] \leq 5\%$.

b. Assume that $\mu(0) = 1$. Find the minimum value of $\mu(1)$ so that $P[\hat{X} = 1|X = 1] \geq 95\%$.