EE126: Problem Set # 8  

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Assigned: October 24, 2003 — Due: October 31, 2003

Problem 1. For $x, y \in \{0, 1\}$, let $P[Y = y \mid X = x] = P(x, y)$ where $P(0, 0) = 1 - P(0, 1) = 0.7$ and $P(1, 1) = 1 - P(1, 0) = 0.6$. Assume that $P(X = 1) = 1 - P(X = 0) = p \in [0, 1]$.

a. Find the MLE of $X$ given $Y$.

b. Find the MAP of $X$ given $Y$.

c. Find the estimate $\hat{X}$ based on $Y$ that minimizes $P[\hat{X} = 0 \mid X = 1]$ subject to $P[\hat{X} = 1 \mid X = 0] \leq \beta$, for $\beta \in (0, 1)$.

Problem 2. Given $X$, the random variables $\{Y_n, n \geq 1\}$ are exponentially distributed with mean $X$. Assume that $P(X = 1) = 1 - P(X = 2) = p \in (0, 1)$.

a. Find the MLE of $X$ given $Y$.

b. Find the MAP of $X$ given $Y$.

c. Find the estimate $\hat{X}_n$ based on $\{Y_1, \ldots, Y_n\}$ that minimizes $P[\hat{X}_n = 1 \mid X = 2]$ subject to $P[\hat{X}_n = 2 \mid X = 1] \leq \beta$, for $\beta \in (0, 1)$.

d. Calculate $P[\hat{X}_n = 1 \mid X = 2]$.

e. Find the value of $n$ so that $P[\hat{X}_n = 1 \mid X = 2] = \alpha$, for some given $\alpha \in (0, 1)$.

Problem 3. Let $X, Y$ be independent random variables where $P(X = -1) = P(X = 0) = P(X = +1) = 1/3$ and $Y$ is $N(0, \sigma^2)$. Find the function $g : \mathbb{R} \to \mathbb{R}$ that minimizes $P(X \neq g(X + Y))$.

Problem 4. Assume that $X$ is uniformly distributed in the set $\{1, 2, 3, 4\}$. When $X = i$, one observes $Y = N(v_i, I)$ where $v_i \in \mathbb{R}^2$ for $i = 1, 2, 3, 4$ and $I$ is the identity matrix in $\mathbb{R}^{2 \times 2}$.

a. Find the function $g : \mathbb{R}^2 \to \{1, 2, 3, 4\}$ that minimizes $P(X \neq g(Y))$.

b. Calculate $P(X \neq g(Y))$ where $g(.)$ was found in part (a).

c. Find vectors $\{v_i, i = 1, 2, 3, 4\}$ that minimize $P(X \neq g(Y))$ subject to $||v_i||^2 \leq 1$ for $i = 1, 2, 3, 4$. 