Communication Networks:
Mathematical Techniques in Networking

EECS 228a

Jean Walrand
University of California
Berkeley
Contents

- Overview
- Graph Theory
- Stochastic Models
- Optimization
- Control Theory
- Game Theory
Overview

- **Topics of course**
  - Not technology (hardware, protocols, products)
  - Mathematical techniques

- **Why such a course?**
  - Technology is easy to learn; many seminars/courses
  - Techniques are usually deep
  - Course objective: panorama/guide to applied mathematics

- **Level of presentations**
  - Intended for researchers
  - Focus on key ideas; illustrate with representative applications
  - Explain intuition behind methods and results
  - Sketch key arguments
  - Indicate references for further reading
Overview (continued)

- **Course format**
  - For each topic: lectures + student presentations
  - Course project: small research project
  - Grading: presentations (65%) and project (35%)

- **Course Material (links on course web site)**
  - Papers/Books
  - Slides of lectures
  - Slides of student presentations
  - Project reports
Overview (continued)

- Quick look at techniques:
  - Graph Theory
  - Stochastic Models
  - Optimization
  - Control Theory
  - Game Theory
Overview/ Graph Theory

- Network as a graph:
  - Network = set of nodes connected by links
  - Each link has
    - capacity (e.g., 100Mbps) and propagation delay
    - reliability
  - Wireless links interfere

- Problems:
  - Routing
    - Shortest path; Path with sufficient capacity
    - Paths to achieve good long-term utilization
    - Path with protection
    - Multicast trees
  - Coloring
    - Scheduling transmissions that conflict (ad hoc network)
    - Assigning channels (cellular network, wi-fi, wi-max, ad-hoc)
  - Matching; Covering
Overview/Graph Theory - Examples

- **Ad Hoc network: Scheduling**
  
  1. **Conflict Graph**
  2. 2 nodes can transmit at a time $\rightarrow$ 40%
  3. Local constraints suggest 50%
  4. Gap between local (cliques) and global

- **Wired network: Routing**
  
  1. Find path from left to right with
     - $\rightarrow$ Delay < 7
     - $\rightarrow$ Cost < 16
  2. -- Complexity of algorithm (P or NP)
  3. -- Good algorithms
Overview/ Stochastic Models

- Source/Channel
  - Entropy Rate/Shannon Capacity → See Information Theory

- Traffic
  - Variable rate: decay of correlation?

- Queuing
  - Predicting backlog and delay
  - Scheduling: Stability

- Network
  - Random network: Connectivity; Small world effects

- User
  - Activity model: depends on network capacity?

- Web Site
  - How are web sites organized?
Overview/ Stochastic Models - Examples

- Sensor Networks: Connectivity

Can information propagate (infinitely) far?

Percolation result:
Yes (w.p.1) iff density is above critical value

- Wired Networks: Connectivity

Number of domains versus out-degree:

\[ n(d) \approx K \times d^{-\alpha} \]

“power law”
Overview/ Stochastic Models - Examples

Model of TCP

- Key Ideas:
  - Sources adjust rate based on observed losses
  - Losses depend on rate of source

- Rate(Losses):
  - Instantaneous rate $x(t)$: AIMD
  - Controlled by “Poisson Losses” ($\lambda$)
  - Analysis $\Rightarrow$ $E(x(t)) = R(\lambda)$

- Losses(Rate):
  - Router queue fed by arrivals
  - Loss rate = increasing in queue length
  - Analysis $\Rightarrow$ $\lambda = L(R)$
Overview/ Optimization

- Routing: Path (lightpaths or flows) selections
  - On/Off line; Myopic or not

- Network Design
  - Capacity Allocation

- Power (Topology) Control

- Techniques: Duality; LP; ILP
Overview/ Optimization - Examples

- **Routing**
  - Ad hoc network
  - Links interfere
  - Requests for flows
  - Find paths to max. accepted flows

- Centralized/Off-line: ILP
- On-line: Heuristic such as minimum interference
- Note that greedy may not work too well
Overview/ Optimization - Examples

- Network

Results:
- For fixed $c$, there is optimum over $R$ and $x$ (may be unstable)
- Optimum over $c$ is hard ... heuristics

$$\max_c \max_R \max_x \sum_j U_j(x_j)$$

subject to $Rx \leq c$ and $\alpha^T x \leq B$
Overview/Control Theory

Internet: distributed feedback control system

- TCP: adapts sending rate to congestion
- AQM: feeds back congestion information

Problem:
- design marking scheme \([q \text{ as function of } y]\)
- design rate adaptation scheme \([x \text{ as function of } q]\)
so that
- system is stable
- flows are “fair”
Overview/ Game Theory

- Routing; Flow Control; Pricing
- Non-Cooperative: Users compete
- Cooperative: Agree to share profits
- Mechanism Design: Design game to promote good outcome
- Price of Anarchy: Social cost of selfish behavior
Overview/ Game Theory - Examples

Flow Control

D → Increases by 1    Increases by 5

A

Increases by 1

Increases by 5

B

C = 50

D

E

(x, y)

22, 22
35, 10
10, 35
15, 15

Too aggressive
→ Losses
→ Throughput falls

One shot: A has an incentive to “cheat”
Long Term: Users have an incentive not to cheat
Overview/ Game Theory - Examples

Routing

Send rate 1 from S to D
- Selfish (each packet chooses cheapest path): SABD $\Rightarrow$ Total cost = 2
- Centralized: 50% SAD, 50% SBD $\Rightarrow$ Total cost = 1.5

Cost of anarchy = 2/1.5 = 4/3 $\Rightarrow$ Extends to linear costs and general topology

Braess Paradox: Remove link AB $\Rightarrow$ Selfish becomes 1.5 (instead of 2)
Routing

Mechanism:
- Each link advertises its cost
- Network computes a “price” per link
- User choose least expensive path

Issue: Design prices so that
- Incentive-compatible: Links advertise their true cost
- Design scalable implementation

Solution:
- Price = reduction in cost of shortest path when link exists (VCG)
- Implementation: BGP-like