Optimal Wireless Protocols and Devices

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Outline

1. **Optimal wireless protocols with guaranteed real-time performance**
   - **Collaborators:** John Doyle and Steven Low

2. **Smart antennas with beamforming, multiplexing and security capabilities**
   - **Collaborators:** Ali Hajimiri, John Doyle and Aydin Babakhani

3. **Very large-scale circuit design: from power circuits to antenna circuits**
   - **Collaborator:** Steven Low
The resource allocation problem is concerned with a fair assignment of rates \( \{x_r\} \) to the users to maximize the utility, under the link capacity and routing constraints.

**Resource allocation problem:**

\[
\max_{x_r \geq 0} \sum_{r \in S} U_r(x_r) \quad \text{s.t.} \quad y_l \leq c_l, \quad \forall l \in \mathcal{L}
\]
To solve this, one can form the corresponding Lagrangian:

\[ L(x, p) = \sum_{r \in S} U_r(x_r) - \sum_{l \in L} p_l(y_l - c_l) \]  

Write the associated KKT conditions:

\[ U'_r(x_r) = q_r, \quad p_l(y_l - c_l) = 0, \quad \forall r \in S, l \in L \]  

Denote the saddle point of the Lagrangian with \((x^*, p^*)\).

**Main idea behind congestion control:** To contrive a distributed control system with a unique stable fixed point \((x^*, p^*)\).
Resource Allocation Problem

- **Drawbacks of current algorithms:**
  - Being oblivious to the transient behavior of the system.
  - Unable to incorporate practical constraints such as buffer size or data size constraints.

- **Objectives of this work:**
  - Design congestion control algorithms with guaranteed real-time performance.
  - Optimal congestion + routing + scheduling...
  - *Particular goal:* Show that the existing algorithms can all be obtained by minimizing appropriate meaningful cost functionals.
  - *Automated synthesis:* Go from utility function maximization to utility functional maximization.
Assume that the following dynamical system exists in the core of the network to generate link prices:

$$\dot{p}_l(t) = h_l(p_l(t))(y_l(t) - c_l)_{p_l(t)}^+, \quad \forall l \in \mathcal{L}$$  \hspace{1cm} (3)

where $p(t)$ and $x(t)$ are the state and the input of the system, respectively.

The objective is to find a cost functional whose minimization leads to the local controllers:

$$x_r(t) = U_r^{r-1}(q_r(t)), \quad \forall r \in S$$  \hspace{1cm} (4)
Theorem

Given $T > 0$, the aforementioned decentralized controller minimizes the cost functional:

$$\min_{x(t)} \left\{ \frac{1}{2} \int_0^T \sum_{l \in \mathcal{L}} \{ Y_l(y_l(t), p_l(t)) + Y_l(\tilde{y}_l(p(t)), p_l(t)) \} dt + \max_{v(T)} L(v(T), p(T)) \right\}$$

where:

$$Y_l(y_l(t), p_l(t)) := (y_l(t) - c_l)h_l(p_l(t))(y_l(t) - c_l)_{+}$$

$$\tilde{y}_l(p(t)) := R \times \arg \max_{v(t)} L(v(t), p(t))$$

Interpretation:

- Terminal cost: counterpart of the static utility function.
- $\tilde{y}_l(p(t))$: optimal instantaneous aggregate rate on link $l$ given the current link price $p(t)$.
- $-Y_l(y_l(t), p_l(t))$: actual $l^{th}$ link utility at time $t$.
- $-Y_l(\tilde{y}_l(p(t), p_l(t))$: virtual $l^{th}$ link utility at time $t$. 
**Corollary**

*For every time instance* \( T > 0 \), *the following relation holds:*

\[
\min_{x(t)} \left\{ \frac{1}{2} \int_0^T \sum_{l \in L} \left\{ Y_l(y_l(t), p_l(t)) + Y_l(\tilde{y}_l(p(t)), p_l(t)) \right\} dt 
+ \max_{v(T)} L(v(T), p(T)) \right\} = \max_{v(0)} L(v(0), p(0))
\]

(6)

- **Getting stability for free!**
- **Optimal routing:**

\[ \max_R \max_{v(0)} L(v(0), p(0)) \] instead of \[ \max_R \min_{p \geq 0} \max_x L(x, p) \] (7)
Dual Algorithm

Is there a simpler cost functional that solves the utility maximization problem?

- Suppose that there exists another cost functional:

\[
\min_{x(t)} \left\{ \int_0^T g(p(t), x(t)) dt + \max_{v(T)} L(v(T), p(T)) \right\}
\]

(8)

that works (there should be some mild assumptions on this functional).

Theorem

There exist a function \( q(p(t), x(t)) \) and a real number \( \mu \) such that:

\[
g(p(t), x(t)) = \mu + q(p(t), x(t)) + \frac{1}{2} \sum_{l \in \mathcal{L}} \left\{ Y_l(y_l(t), p_l(t)) + Y_l(\tilde{y}_l(p(t)), p_l(t)) \right\}
\]

(9)

where the function \( q(p(t), x(t)) \) is zero along all trajectories of the optimal closed-loop system.
Real-Time Performance

- Similar story for primal and primal/dual algorithms...
- Several extensions are possible!
- One can see that the dynamics that the dual algorithm maximizes is not really what we want in reality.
- A question arises as to how to refine the existing congestion controllers to obtain a satisfactory real-time performance.
Design Specifications

Desired Steady-state behavior:

1) $x_r(t)$ converges to $x_r^*$ as $t$ goes to infinity.

Desired transient behavior:

1) The system is sufficiently robust to delays.
2) The rate of every user is non-oscillatory.
3) $y_l(t)$ never exceeds a pre-assigned virtual capacity $\tilde{c}_l$.
4) $x_r(t)$ increases if its route is not congested, i.e. if $y_l(t)$ is less than $c_l - \Delta c_r$ for every $l \in \mathcal{L}(r)$. 
Virtual Capacity

- We propose a set of congestion controllers parameterized in terms of $\alpha$.
- We have a parameterized utility functional.

**Definition**

Define the virtual capacity $\tilde{c}_l(\alpha)$ as the smallest number greater than $c_l$ for which the inequality

$$2|S(l)| \left( \max_{x \in [0,y_l], r \in S(l)} k(x)U'_r(x) \right)^2 \leq (y_l - c_l)^2 k \left( \frac{y_l}{|S(l)|} \right) hw_\alpha (y_l - c_l)$$

(10)

**Definition**

Define the residual capacity associated with source $r$ as:

$$\Delta c_r(\alpha, \mathbf{x}(0), \mathbf{p}(0)) := -w_\alpha^{-1} \left( \frac{U'_r(\min_{l \in \mathcal{L}(r)} c_l)}{\sqrt{2h|\mathcal{L}(r)| V(\mathbf{x}(0), \mathbf{p}(0)) + q_r^*}} \right)$$

(11)
Guaranteed Real-Time Performance

Theorem

i) If $y_l(0)$ is less than or equal to $c_l$, then $y_l(t)$ can never exceed the virtual capacity $\tilde{c}(\alpha)$.

ii) If $y_l(0)$ is greater than $c_l$, then $y_l(t)$ will monotonically decrease until it becomes less than or equal to the virtual capacity $\tilde{c}_l(\alpha)$, provided:

$$\tilde{p}_l(0) \geq \frac{h(\tilde{c}_l(\alpha) - c_l)^2}{2|S(l)| \max_{x \in [0, \tilde{c}_l(\alpha)], r \in S(l)} k(x) U'_r(x)}$$  \hspace{1cm} (12)

iii) $x_r(t)$ increases if $y_l(t)$ is less than $c_l - \Delta c_r(\alpha, x(0), \tilde{p}(0))$ for all $l \in \mathcal{L}(r)$.

A similar theorem for the delay case after a slight modification...
Numerical Example

\[ X_1 \quad 1 \quad 2 \quad X_2 \quad 3 \quad 4 \]

Link 1 \quad Link 2 \quad Link 3

\[ X_3 \]

\[ x_1(t) \]

\[ x_2(t) \]

\[ x_3(t) \]

\[ y_1(t) \]

\[ y_2(t) \]

\[ \text{Price for Link 1} \]
Antennas

- **Antenna**: A transducer to transmit or receive electromagnetic waves.
- Antennas convert electrical current into electromagnetic radiation, or vice versa.
- **Radiation Pattern**: The variation of the power radiated by an antenna as a function of the direction away from the antenna.
Conventional antennas for wireless transmission, e.g. omni-directional antennas, radiate in almost all directions.
A great amount of effort has been made in the past several decades to design smart transmitting/receiving antenna systems in order to:

- Have a better security
- Avoid co-channel interference
- Avoid unnecessary power consumption in undesired directions
Array Antennas

- An array system comprises multiple active (antenna) elements for varying the relative phases and amplitudes of the respective signals in order to generate a desired radiation pattern.

- Array antennas are easy to program because the individual radiation patterns add linearly.
Array Antennas

- $L_r$: Size of the array divided by the wavelength; $n_r$: Number of antenna elements.

Array antennas are easy to program but hard to implement due to the distance between the antenna elements being in the order of the wavelength!
Passive Antennas

- Passive antennas mainly use one active element with several passive (parasitic) elements to shape the radiation pattern.

- Benefits: chip-size reduction, less power consumption, no stability issue...
Passive Antennas

- Passive antennas are easy to implement but hard to program due to not being clear how to design the passive elements (the linearity property of the array antennas does not hold).

- To generate the following not-so-good patterns, the design of the antenna has taken more than 4 weeks (running time)!
Objective

- Using combining theories from algebraic geometry and convex optimization the goal is to design a passively controllable smart (PCS) antenna such that
  - It can be implemented as an integrated circuit (size = in the order of one wavelength).
  - It can be programmed for real-time data transmission in arbitrary directions.

**Example:** Send data from node 0 to node 1 using a PCS antenna so that all other nodes receive a zero signal!
Passively Controllable Smart Antennas

- A PCS antenna is composed of only one active element, some reflectors, and several controllable passive elements.
- The controllable ports can potentially shape the radiation pattern!
- The following antenna system can potentially generate $2^{12}$ different voltages at the upright direction.
Passively Controllable Smart Antennas

- A PCS antenna can send independent data to different directions.

- A prototype of a PCS antenna system:

- The main challenge is how to control the controllable ports so that a desired radiation pattern is created.
In this talk, we show that a PCS antenna integrates all benefits of array and passive antennas.

For example, we can design superior radiation patterns in a few seconds!

Excellent beamforming capability!
Consider a PCS antenna radiating in $z$ directions, which has $n - z$ controllable ports.

Linear network block: scattering matrix
- $v_1, \ldots, v_z$: voltages seen at different directions.
- $v_{z+1}, \ldots, v_n$: voltages on the reflectors (to be controlled).

Control unit: a programmable passive controller on the antenna.
Passively Controllable Smart Antennas

**Goal:** Identify the space $D$ consisting of all possible radiation vectors $(v_1, v_2, ..., v_z)$ under a passive controller.

- The space $D$ can be used to characterize:
  - the set of all voltages that can be sent to any direction;
  - the possible correlation between signals sent in different directions.

- To program a PCS antenna, we need to know:
  - how to find the feasibility region $D$;
  - how to design a controller corresponding to each point in $D$.

**Intermediate Step:** In practice, the controller of a PCS antenna is preferred to be decentralized, but for now we try to identify the space of all possible radiation vectors $(v_1, v_2, ..., v_z)$ under a centralized passive controller.
Theorem

Given \( \alpha \in \mathbb{C}^{1 \times z} \), the PCS antenna can be programmed (via a passive centralized controller) to generate the pattern \((v_1, v_2, \ldots, v_z) = \alpha\) if and only if there exist symmetric matrices \(M, N \in \mathbb{R}^{(n-z) \times (n-z)}\) such that

\[
\begin{bmatrix}
(\text{Re}\{W_{22} - W_{21}W_{11}^{-1}W_{12}\})^{-1} - M & N \\
N & M
\end{bmatrix} \succ 0,
\]

and

\[
-(W_{31}W_{11}^{-1}W_{12} - W_{32})(M + Ni)W_{21}W_{11}^{-1} - W_{31}W_{11}^{-1} = \alpha.
\]

- The feasibility region \(D\) for the radiation pattern of a PCS antenna is given by (1) and (2).
- The constraints (1) and (2) are linear in the variables \(M\) and \(N\).
Lemma

Given a scalar $m \in \mathbb{N}$ and vectors $x_1, x_2 \in \mathbb{R}^{1 \times m}$ with the property $\| \begin{bmatrix} x_1 & x_2 \end{bmatrix} \| = 1$, consider the set of all vectors $\alpha \in \mathbb{R}^{1 \times 2m}$ for which there exist symmetric matrices $M, N \in \mathbb{R}^{m \times m}$ such that

$$\alpha = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} M & N \\ N & -M \end{bmatrix}$$ (15)

and

$$\begin{bmatrix} M & N \\ N & -M \end{bmatrix} \prec I.$$ (16)

This set is identical to the open unit ball $\{ \gamma \in \mathbb{R}^{1 \times 2m} \mid \| \gamma \| < 1 \}$.

- The challenging part of the proof is “given $\alpha$ in the ball, how to find $M$ and $N$ satisfying the constraints (3) and (4)?”
Given scalars \( m, k \in \mathbb{N} \), vectors \( \mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^{1 \times m} \) and matrices \( G_1, G_2 \in \mathbb{R}^{m \times k} \), consider the set of all complex vectors \( \alpha \in \mathbb{C}^{1 \times k} \) that can be written as

\[
\alpha = (\mathbf{x}_1 + \mathbf{x}_2 i)(M + Ni)(G_1 + G_2 i)
\]

for some symmetric matrices \( M, N \in \mathbb{R}^{m \times m} \) with the property

\[
\begin{bmatrix}
  M & N \\
  N & -M
\end{bmatrix} \prec I.
\]

The real-valued representation of this complex set is identical to the ellipsoid

\[
\left\{ \mathbf{h} \in \mathbb{R}^{1 \times 2k} \mid \mathbf{h} \left( \begin{bmatrix}
  G_1^* & -G_2^* \\
  G_2^* & G_1^*
\end{bmatrix} \begin{bmatrix}
  G_1 & G_2 \\
  -G_2 & G_1
\end{bmatrix} \right)^{-1} \mathbf{h}^* < \| \mathbf{x}_1 \|^2 + \| \mathbf{x}_2 \|^2 \right\}.
\]

\[\text{Lemma}\]
Theorem

A pattern \((v_1, v_2, ..., v_z) = \alpha\) can be generated via the passive centralized control of the PCS antenna if and only if \([\text{Re}\{\alpha\} \quad \text{Im}\{\alpha\}]\) belongs to the ellipsoid

\[
\left\{ \begin{array}{c}
h \in \mathbb{R}^{1 \times 2z} \\
(h - o)\Omega^{-1}(h - o)^* < \frac{1}{4}\|K_1 Q^{1/2}\|^2
\end{array} \right\}. \tag{19}
\]

\[
K_1 := W_{31} W_{11}^{-1} W_{12} - W_{32}, \quad K_2 := W_{21} W_{11}^{-1} W_{13} - W_{23},
\]
\[
K_3 := W_{31} W_{11}^{-1} W_{13} - W_{33}, \quad K_4 := W_{21} W_{11}^{-1}, \quad K_5 := W_{31} W_{11}^{-1},
\]
\[
Q := (\text{Re}\left\{ W_{22} - W_{21} W_{11}^{-1} W_{12} \right\})^{-1},
\]
\[
o := - \left[ \begin{array}{c}
\text{Re}\left\{ \frac{1}{2} K_1 Q K_4 + K_5 \right\} \\
\text{Im}\left\{ \frac{1}{2} K_1 Q K_4 + K_5 \right\}
\end{array} \right],
\]
\[
\Omega := \left[ \begin{array}{cc}
\text{Re}\{K_4^* Q K_4\} & \text{Im}\{K_4^* Q K_4\} \\
-\text{Im}\{K_4^* Q K_4\} & \text{Re}\{K_4^* Q K_4\}
\end{array} \right].
\]
Passively Controllable Smart Antennas

- **Objective 1**: Maximize the received power in the direction $\theta = 90^\circ$
- **Objective 2**: Maximize the received power in the direction $\theta = 45^\circ$
- **Objective 3**: Maximize the received power in the direction $\theta = 90^\circ$ so that no signal is sent to all of the directions $15^\circ, 30^\circ, 45^\circ, 60^\circ, 120^\circ, 135^\circ, 150^\circ, 165^\circ$
PCS Antennas: Decentralized Case

- Nice ellipsoidal feasibility region at the passive, centralized case!
- The hope is that this region is mostly a consequence of passivity, rather than the centralized control.

- There are an exponential number of possibilities for the signs of current and voltages; but we use a randomization algorithm...
PCS Antennas: Decentralized Case

![Diagram of PCS Antennas: Decentralized Case](image)

- Reflectors and controllable ports
- Dipole Antenna

![Complex Plane Plots](image)
Optimal Operating Point of a Circuit

- Antenna design:

- Optimal power flow problem:
Since 1962, numerous algorithms have been adapted for OPF:
- Linear programming
- Interior point method
- Nonlinear programming
- Dynamic programming
- Lagrangian relaxation
- Decomposition method
- Genetic algorithms...

We obtained a condition under which OPF is easy to solve!
- The condition is satisfied for IEEE benchmark systems with 14, 30, 57, 118 and 300 buses.
- The main reason is the non-negativity of physical quantities.

Our ideas can be generalized to VLSI and antenna design...
Concluding Remarks

- We studied the design of optimal protocols and optimal devices in wireless networks.

- We combined different theories from control, network, circuit, communication, and partially power.

- For the first time, we designed an on-chip antenna whose size is about 15 times smaller than the existing ones.

- We reduced the running time of the design from 4 weeks to 2 seconds...

- Our results have been published in different communities: INFOCOM, GLOBECOM, Allerton, Antenna Propagation, ACC, CDC, MTNS, Circuit...