Multipath and Power Laws

Wei Wei
10/14/2010
MURI 2010
Outline

- Can Multipath Mitigate Power Law Delays?
  - Jian Tan, Wei Wei, Bo Jiang, Ness Shroff, Don Towsley, Sigmetrics 2010, 22nd International Teletraffic Congress (ITC 22)

- Stochastic Differential Equations for Power Law Behaviors
  - Bo Jiang, Roger Brockett, Weibo Gong, Don Towsley, submitted to Journal of Applied Probability
Can Multipath Mitigate Power Law Delays?  
-- Effects of Parallelism on Tail Performance

Jian Tan*, Wei Wei+, Bo Jiang+, Ness Shroff*,  
Don Towsley+

*The University of Ohio State  
+University of Massachusetts, Amherst
Outline

- Motivation
- Redundant transmission
- Split transmission
- Optimal split
- Conclusions
Motivation: Effect of Parallelism on Tail Behavior

- Parallelism common approach to improve reliability, efficiency
  - peer to peer systems
  - grid computing
  - multipath in communication networks
- Job failures lead to power law tails in sequential systems

Can parallelism mitigate power law tails?
Motivation - Outages Lead to Power Law Retransmissions

- Packet length $L$: $F(x) = P(L > x)$
- On-off channel: $A$, $U$ $G(x) = P(A > x)$

- If $L > A_i$, retransmit entire packet
- $N$: # of transmissions needed to deliver a packet

- If $\lim_{x \to \infty} \frac{\log F(x)}{\log G(x)} = \alpha$, then $\lim_{n \to \infty} \frac{\log P(N > n)}{\log n} = -\alpha$.
  - Jelenkovic & Tan, Infocom 2007

- Example:
  $$F(x) = e^{-\alpha x}, \quad G(x) = e^{-x}$$

**Light tail distributions can lead to power laws**
Motivation - Retransmissions Lead to Power Law Delays

T: packet delivery time

\[
T = \sum_{i=1}^{N-1} (A_i + U_i) + L
\]

If \( \lim_{x \to \infty} \frac{\log \overline{F}(x)}{\log G(x)} = \alpha \), and \( E[(A + U)^{1+\alpha+\theta}] < \infty \),

for some \( \theta > 0 \), then \( \lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\alpha \).

- Jelenkovic & Tan, Infocom 2007
Can Multipath Mitigate Power Law Delays?

- Given $K$ independent channels
  - redundant transmission
    - duplicate packet, send over $K$ channels
  - split transmission
    - split packet into $K$ pieces, send over $K$ channels

- Questions
  - can either reduce delay? when?
  - heterogeneous channels?
Redundant Transmissions – homogeneous paths

- Success if transmission on one channel succeeds
  \[ T = \min\{T_1, T_2, \ldots, T_K\} \]

- If \( \lim_{x \to \infty} \frac{\log \overline{F}(x)}{\log G(x)} = \alpha, E[L^{\alpha+\theta}] < \infty, E[U^{\max\{1,\alpha\}+\theta}] < \infty, E[A^{1+\theta}] < \infty \)
  for some \( \theta > 0 \), then \( \lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\alpha. \)

Redundant transmission does not mitigate power law delays
Redundant Transmission – Heterogeneous Paths

- Heterogeneous paths
  - if \( \lim_{x \to \infty} \frac{\log F(x)}{\log G_j(x)} = \alpha_j, j = 1 \ldots K \),
  
  and conditions on moments of \( L, A_j, U_j \)

  then \( \lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\max\{\alpha_j\} \).

- Delay determined by best path
- Can derive exact asymptotic distribution
Split Transmission

- Tradeoff
  - smaller packets in channels
  - but
  - success required on all channels

\[ T = \max\{T_1, T_2, \ldots, T_K\} \]

- General results as for redundant transmission?
- Dependence on F and G?
Split Transmission – Homogeneous Paths

If
\[
\lim_{x \to \infty} \frac{\log F(x)}{\log G(x)} = \alpha, \quad \lim_{x \to \infty} \frac{\log F(x)}{\log F(x/K)} = \beta,
\]

and moment conditions on \( L, A, U \)

then
\[
\lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\beta \alpha.
\]

for split transmission over \( K \) i.i.d. channels.

\( \beta \) always \( \geq 1 \) \( \Rightarrow \) split transmission no worse than single path transmission.

Exponential packet lengths: \( \beta = K \), Pareto: \( \beta = 1 \),

Weibull: \( \beta = K^b \)

*General rule behind examples?
When is Split Beneficial?

• **Terminology**
  
  positive function $f$ **regularly varying** at infinity with index $\rho$ if
  
  $$
  \lim_{x \to \infty} \frac{f(\lambda x)}{f(x)} = \lambda^\rho, \quad \forall \lambda > 0
  $$
  
  $f$ **slowly varying** if $\rho = 0$

  
• **Benefits of split**
  
  • split transmission no worse than single path transmission
  
  • not beneficial when
    
    $$
    \beta = \lim_{x \to \infty} \frac{\log \bar{F}(x)}{\log \bar{F}(x / K)} = 1,
    $$
    
  i.e., $-\log \bar{F}(x)$ **slowly varying**.

  • beneficial when $\beta > 1$, i.e., $-\log \bar{F}(x)$ **regularly varying** with positive index
Split Transmission – Heterogeneous Paths

T: transmission delay

If \( \lim_{x \to \infty} \frac{\log F(x)}{\log G_j(x)} = \alpha_j, \quad \lim_{x \to \infty} \frac{\log F(x)}{\log F(\gamma_j x)} = \beta_j, \)

and conditions on moments of \( L, U_j, A_j \)

then \( \lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\min\{\beta_j \alpha_j\}, \)

for split transmission over \( K \) independent channels.
Optimal Split

If \( \beta(\gamma) = \lim_{x \to \infty} \frac{\log F(x)}{\log F(\gamma x)} \) exists, then \( \beta(\gamma) = \gamma^{-\rho} \).

If \( \rho > 0 \), then \( \gamma_j^* = \frac{\alpha_j^{1/\rho}}{\sum_{i=1}^K \alpha_i^{1/\rho}} \), \( \lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\left( \sum_{i=1}^K \alpha_i^{1/\rho} \right)^\rho \).

If \( \rho = 0 \), then arbitrarily split among best paths, and
\[
\lim_{t \to \infty} \frac{\log P(T > t)}{\log t} = -\max \{ \alpha_j \}.
\]

- Optimal split outperforms redundant transmission
- Exact asymptotic results attainable
Conclusions

- Investigated whether and how parallelism improves network performance
- Redundant transmission does not mitigate power law delays
- Split transmission mitigates power delays
  - if absolute value of logarithm of packet size probability tail is regularly varying with positive index
- Optimal split outperforms redundant transmission
- Results can be extended to many other applications involving parallelism, job failures
  - computing jobs in grid computing
  - file downloading in peer to peer networks
  - parallel experiment planning, parallel scheduling
Sample Path and Density (Weibull)

- Sub-linear Preferential Attachment

The sample path of $dX_t = \mu X_t^\delta dt + (X_0 - X_t)dN_t$ is plotted against $t$. The density of the Weibull distribution is shown on a log-log scale against $x$. The parameters are:
- $\mu = 5$
- $\lambda = 3$
- $X_0 = 1$
- $\delta = 0.5$
Optimal Split - Examples

- **Exponential:**
  \[ F(x) = P(L > x) = e^{-\lambda x}, \quad G_j = P(A^j > x) = e^{-\mu_j x} \]
  \[ \alpha_j = \frac{\log F(x)}{\log G_j(x)} = \frac{\lambda}{\mu_j}, \quad \beta(\gamma) = \frac{\log F(x)}{\log F(\gamma x)} = 1 / \gamma \]

  \[ \rho = -\log \beta(\gamma) / \log \gamma = 1, \quad \gamma_j = \frac{1 / \mu_j}{\sum_{i=1}^{K} 1 / \mu_i} \]

- **Weibull:**
  \[ F(x) = P(L > x) = e^{-(\lambda x)^b}, \quad G_j = P(A^j > x) = e^{-(\mu_j x)^b} \]
  \[ \alpha_j = \frac{\log F(x)}{\log G_j(x)} = \left( \frac{\lambda}{\mu_j} \right)^b, \quad \beta(\gamma) = \frac{\log F(x)}{\log F(\gamma x)} = 1 / \gamma^b \]

  \[ \rho = -\log \beta(\gamma) / \log \gamma = b, \quad \gamma_j = \frac{1 / \mu_j}{\sum_{i=1}^{K} 1 / \mu_i} \]

- **Pareto:** \( \rho = 0, \) split among best paths